## Chapter 12, Solution 1.

$$m = 20 \text{ kg}, \; g = 3.75 \text{ m/s}^2$$

$$W = mg = (20)(3.75) \qquad W = 75 \text{ N} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 2.**

At all latitudes, $m = 2.000 \text{ kg}$ ◀

$(a)$ $\phi = 0°$, $g = 9.7807\left(1 + 0.0053 \sin^2 \phi\right) = 9.7807 \text{ m/s}^2$

$$W = mg = (2.000)(9.7807) \qquad W = 19.56 \text{ N} \blacktriangleleft$$

$(b)$ $\phi = 45°$, $g = 9.7807\left(1 + 0.0053 \sin^2 45°\right) = 9.8066 \text{ m/s}^2$

$$W = mg = (2.000)(9.8066) \qquad W = 19.61 \text{ N} \blacktriangleleft$$

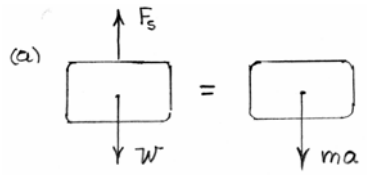
$(c)$ $\phi = 60°$, $g = 9.7807\left(1 + 0.0053 \sin^2 60°\right) = 9.8196 \text{ m/s}^2$

$$W = mg = (2.000)(9.8196) \qquad W = 19.64 \text{ N} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 3.

Assume $g = 32.2$ ft/s$^2$




$$m = \frac{W}{g}$$

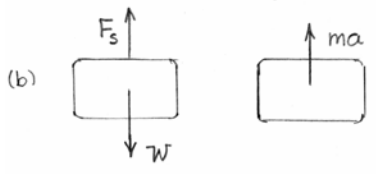
$$+\downarrow \Sigma F = ma: \quad W - F_s = \frac{W}{g}a$$

$$W\left(1 - \frac{a}{g}\right) = F_s \qquad \text{or} \qquad W = \frac{F_s}{1 - \frac{a}{g}} = \frac{7}{1 - \frac{2}{32.2}}$$

$$W = 7.46 \text{ lb} \blacktriangleleft$$

$$m = \frac{W}{g} = \frac{7.4635}{32.2} = 0.232 \text{ lb}\cdot\text{s}^2/\text{ft}$$

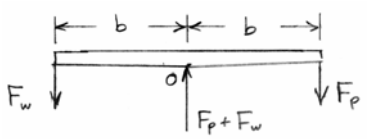
$$\Sigma F = ma: \quad F_s - W = \frac{W}{g}a$$




$$F_s = W\left(1 + \frac{a}{g}\right)$$

$$= 7.46\left(1 + \frac{2}{32.2}\right) \qquad F_s = 7.92 \text{ lb} \blacktriangleleft$$

For the balance system *B*,




$$\Sigma M_0 = 0: \quad bF_w - bF_p = 0$$

$$F_w = F_p$$

But, $F_w = W_w\left(1 + \frac{a}{g}\right)$ and $F_p = W_p\left(1 + \frac{a}{g}\right)$

so that $W_w = W_p$ and $m_w = \frac{W_p}{g}$ $\qquad m_w = 0.232 \text{ lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 4.

Periodic time: $\tau = 12 \text{ h} = 43200 \text{ s}$

Radius of Earth: $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

Radius of orbit: $r = 3960 + 12580 = 16540 \text{ mi} = 87.33 \times 10^6 \text{ ft}$

Velocity of satellite: $$v = \frac{2\pi r}{\tau} = \frac{(2\pi)\left(87.33 \times 10^6\right)}{43200}$$

$$= 12.7019 \times 10^3 \text{ ft/s}$$

It is given that $mv = 750 \times 10^3 \text{ lb}\cdot\text{s}$

(*a*) $$m = \frac{mv}{v} = \frac{750 \times 10^3}{12.7019 \times 10^3} = 59.046 \text{ lb}\cdot\text{s}^2/\text{ft}$$

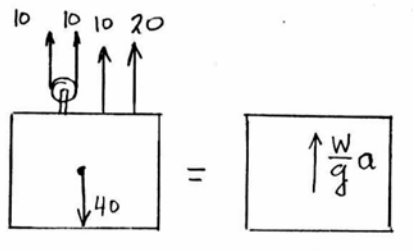
$m = 59.0 \text{ lb}\cdot\text{s}^2/\text{ft}$ ◀

(*b*) $$W = mg = (59.046)(32.2) = 1901 \text{ lb}$$

$W = 1901 \text{ lb}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 5.




$$+\uparrow \sum F_y = ma_y: \qquad 10 + 10 + 10 + 20 - 40 = \frac{40}{32.2} a_y$$

$$a_y = \frac{(32.2)(10)}{40} = 8.05 \text{ ft/s}^2 \uparrow$$

$$a_y = \frac{dv}{dt} = \frac{dy}{dt}\frac{dv}{dy} = v\frac{dv}{dy}$$

$$v\,dv = a_y\,d_y$$

$$\int_0^v v\,dv = \int_0^v a_y\,d_y \qquad \frac{1}{2}v^2 = a_y y$$

$$v = \sqrt{2a_y y} = \sqrt{(2)(8.05)(1.5)} \qquad v = 4.91 \text{ ft/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 6.

Data: $v_0 = 108$ km/h $= 30$ m/s, $x_f = 75$ m

(*a*) Assume constant acceleration. $a = v\dfrac{dv}{dx} = \dfrac{dv}{dt} = \text{constant}$

$$\int_{v_0}^{0} v\,dv = \int_{0}^{x_f} a\,dx$$

$$-\frac{1}{2}v_0^2 = a\,x_f$$

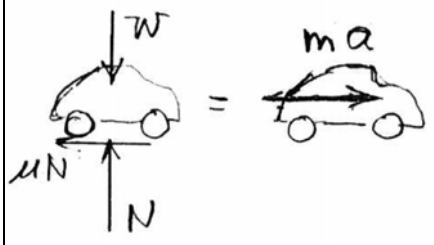
$$a = -\frac{v_0^2}{2x_f} = -\frac{(30)}{(2)(75)} = -6 \text{ m/s}^2$$

$$\int_{v_0}^{0} dv = \int_{0}^{t_f} a\,dt$$

$$-v_0 = a\,t_f$$

$$t_f = -\frac{v_0}{a} = \frac{-30}{-6} \qquad t_f = 5.00 \text{ s} \blacktriangleleft$$

(*b*)




$$+\uparrow \sum F_y = 0: \quad N - W = 0$$

$$N = W$$

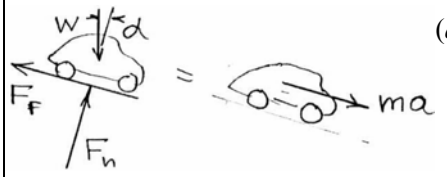
$$\pm \sum F_x = ma: \qquad -\mu N = ma$$

$$\mu = -\frac{ma}{N} = -\frac{ma}{W} = -\frac{a}{g}$$

$$\mu = -\frac{(-6)}{9.81} \qquad \mu = 0.612 \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 7.




(*a*) $+\nearrow \sum F = ma$: $\quad -F_f + W\sin\alpha = ma$

$$a = -\frac{F_f}{m} + \frac{W\sin\alpha}{m} = -\frac{F_f}{m} + g\sin\alpha$$

$$= -\frac{7500\text{ N}}{1400\text{ kg}} + \left(9.81\text{ m/s}^2\right)\sin 4^\circ = -4.6728\text{ m/s}^2$$

$$a = 4.6728\text{ m/s}^2 \searrow 4^\circ$$

$$v_0 = 88\text{ km/h} = 24.444\text{ m/s}$$

From kinematics, $\quad a = v\dfrac{dv}{dx}$

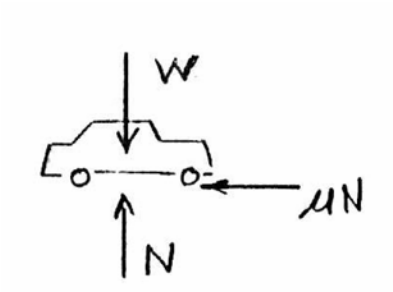
$$\int_0^{x_f} a\,dx = \int_{v_0}^{0} v\,dv$$

$$a\,x_f = -\frac{1}{2}v_0^2$$

$$x_f = -\frac{v_0^2}{2a} = -\frac{(24.444)^2}{(2)(-4.6728)} \qquad x_f = 63.9\text{ m} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 8.




(*a*) Coefficient of static friction.

$$\Sigma F_y = 0: \qquad N - W = 0 \qquad N = W$$

$$v_0 = 70 \text{ mi/h} = 102.667 \text{ ft/s}$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a_t\left(s - s_0\right)$$

$$a_t = \frac{v^2 - v_0^2}{2\left(s - s_0\right)} = \frac{0 - \left(102.667\right)^2}{\left(2\right)\left(170\right)} = -31.001 \text{ ft/s}^2$$

For braking without skidding $\mu = \mu_s$, so that $\mu_s N = m|a_t|$

$$\stackrel{+}{\rightarrow} \Sigma F_t = ma_t: \quad -\mu_s N = ma_t$$

$$\mu_s = -\frac{ma_t}{W} = -\frac{a_t}{g} = \frac{31.001}{32.2} \qquad \mu_s = 0.963 \blacktriangleleft$$

(*b*) Stopping distance with skidding.

Use $\mu = \mu_k = \left(0.80\right)\left(0.963\right) = 0.770$

$$\stackrel{+}{\leftarrow} \Sigma F = ma_t: \quad \mu_k N = -ma_t$$

$$a_t = -\frac{\mu_k N}{m} = -\mu_k g = -24.801 \text{ ft/s}^2$$

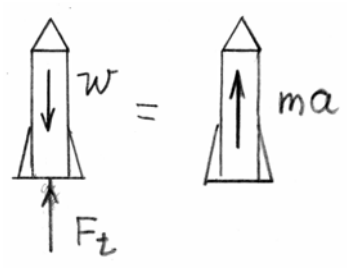
Since acceleration is constant,

$$\left(s - s_0\right) = \frac{v^2 - v_0^2}{2a_t} = \frac{0 - \left(102.667\right)^2}{\left(2\right)\left(-24.801\right)}$$

$$s - s_0 = 212 \text{ ft} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 9.




For the thrust phase, $+\uparrow \Sigma F = ma$: $F_t - W = ma = \dfrac{W}{g}a$

$$a = g\left(\frac{F_t}{W} - 1\right) = (32.2)\left(\frac{2}{0.2} - 1\right) = 289.8 \text{ ft/s}^2$$

At $t = 1$ s,

$$v = at = (289.8)(1) = 289.8 \text{ ft/s}$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}(289.8)(1)^2 = 144.9 \text{ ft}$$

For the free flight phase, $t > 1$ s. $a = -g = -32.2$ ft/s

$$v = v_1 + a(t-1) = 289.8 + (-32.2)(t-1)$$

At $v = 0$, $\quad t - 1 = \dfrac{289.8}{32.2} = 9.00$ s, $\quad t = 10.00$ s

$$v^2 - v_1^2 = 2a(y - y_1) = -2g(y - y_1)$$

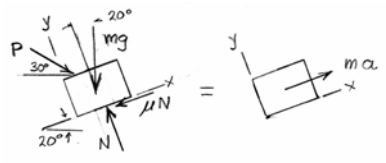
$$y - y_1 = -\frac{v^2 - v_1^2}{2g} = -\frac{0 - (289.8)^2}{(2)(32.2)} = 1304.1 \text{ ft}$$

(*a*) $y_{\max} = h = 1304.1 + 144.9$ $\qquad h = 1449$ ft ◀

(*b*) As already determined, $\qquad t = 10.00$ s ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 10.




Kinematics: Uniformly accelerated motion. $(x_0 = 0,\ v_0 = 0)$

$$x = x_0 + v_0 t + \frac{1}{2}at^2, \qquad \text{or} \qquad a = \frac{2x}{t^2} = \frac{(2)(10)}{(4)^2} = 1.25 \text{ m/s}^2$$

$$+\nwarrow \Sigma F_y = 0: \quad N - P\sin 50° - mg\cos 20° = 0$$

$$N = P\sin 50° + mg\cos 20°$$

$$+\nearrow \Sigma F_x = ma: \quad P\cos 50° - mg\sin 20° - \mu N = ma$$

$$\text{or} \quad P\cos 50° - mg\sin 20° - \mu\left(P\sin 50° + mg\cos 20°\right) = ma$$

$$P = \frac{ma + mg\left(\sin 20° + \mu\cos 20°\right)}{\cos 50° - \mu\sin 50°}$$

For motion impending, set $a = 0$ and $\mu = \mu_s = 0.30$.

$$P = \frac{(40)(0) + (40)(9.81)(\sin 20° + 0.30\cos 20°)}{\cos 50° - 0.30\sin 50°} = 593 \text{ N}$$

For motion with $a = 1.25 \text{ m/s}^2$, use $\mu = \mu_k = 0.25$.

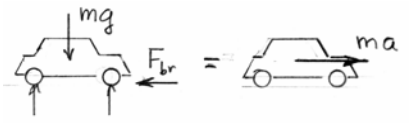
$$P = \frac{(40)(1.25) + (40)(9.81)(\sin 20° + 0.25\cos 20°)}{\cos 50° - 0.25\sin 50°}$$

$$P = 612 \text{ N} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 11.

Calculation of braking force/mass $\left(F_b/m\right)$ from data for level pavement.




$$v_0 = 100 \text{ km/hr} = 27.778 \text{ m/s}$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a\left(x - x_0\right)$$

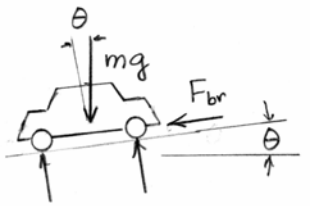
$$a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left(27.778\right)^2}{\left(2\right)\left(60\right)}$$

$$= -6.43 \text{ m/s}^2$$

$$\Sigma F_x = ma: \quad -F_{\text{br}} = ma$$

$$\frac{F_{\text{br}}}{m} = -a = 6.43 \text{ m/s}^2$$

(*a*) Going up a 6° incline. $\left(\theta = 6°\right)$




$$+\nearrow \Sigma F = ma: \quad -F_{\text{br}} - mg\sin\theta = ma$$

$$a = -\frac{F_{\text{br}}}{m} - g\sin\theta$$

$$= -6.43 - 9.81\sin 6° = -7.455 \text{ m/s}^2$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(27.778\right)^2}{\left(2\right)\left(-7.455\right)}$$

$$x - x_0 = 51.7 \text{ m} \blacktriangleleft$$

(*b*) Going down a 2% incline. $\left(\tan\theta = -0.02,\ \theta = -1.145°\right)$

$$+\nearrow \Sigma F = ma: \quad -F_{\text{br}} - mg\sin\theta = ma$$

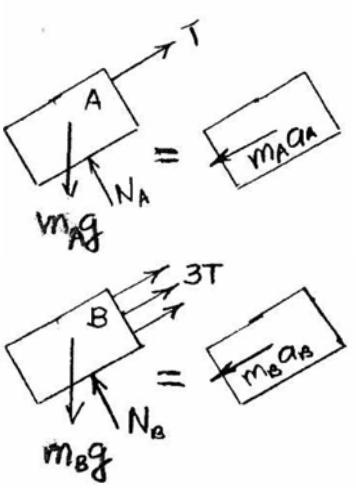
$$a = -\frac{F_{\text{br}}}{m} - g\sin\theta$$

$$= -6.43 - 9.81\sin\left(-1.145°\right) = -6.234 \text{ m/s}^2$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(27.778\right)^2}{\left(2\right)\left(-6.234\right)} \qquad x - x_0 = 61.9 \text{ m} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 12.**




Let the positive directions of $x_A$ and $x_B$ be down the incline.

Constraint of the cable: $x_A + 3x_B = \text{constant}$

$$a_A + 3a_B = 0 \qquad \text{or} \qquad a_B = -\frac{1}{3}a_A$$

For block $A$: $+\nearrow \Sigma F = ma$: $m_A g \sin 30° - T = m_A a_A$ (1)

For block $B$: $+\nearrow \Sigma F = ma$: $m_B g \sin 30° - 3T = m_B a_B = -m_B a_A$ (2)

Eliminating $T$ and solving for $\dfrac{a_A}{g}$,

$$\left(3m_A g - m_B g\right)\sin 30° = \left(3m_A + \frac{m_B}{3}\right)a_A$$

$$\frac{a_A}{g} = \frac{\left(3m_A - m_B\right)\sin 30°}{3m_A + m_B/3} = \frac{\left(30 - 8\right)\sin 30°}{30 + 2.667} = 0.33673$$

(*a*) $a_A = (0.33673)(9.81) = 3.30 \text{ m/s}^2$ $\qquad \mathbf{a}_A = 3.30 \text{ m/s}^2 \measuredangle 30°$ ◀

$$a_B = -\frac{1}{3}(3.30) = -1.101 \text{ m/s}^2 \qquad \mathbf{a}_B = 1.101 \text{ m/s}^2 \measuredangle 30°$$ ◀

(*b*) Using equation (1),

$$T = m_A g\left(\sin 30° - \frac{a_A}{g}\right) = (10)(9.81)(\sin 30° - 0.33673)$$

$$T = 16.02 \text{ N}$$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 13.

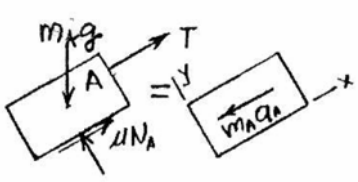
Let the positive directions of $x_A$ and $x_B$ be down the incline.

Constraint of the cable: $x_A + 3x_B = \text{constant}$

$$a_A + 3a_B = 0 \qquad a_B = -\frac{1}{3}a_A$$

Block $A$: $+\nwarrow \Sigma F_y = 0$: $N_A - m_A g\cos 30° = 0$

$$+\swarrow \Sigma F_x = ma: \quad m_A g\sin 30° - \mu N_A - T = m_A a_A$$

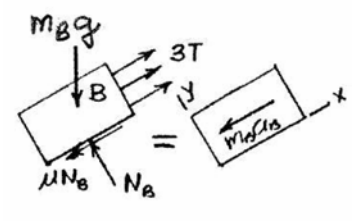



Eliminate $N_A$.

$$m_A g\left(\sin 30° - \mu\cos 30°\right) - T = m_A a_A$$

Block $B$: $+\nwarrow \Sigma F_y = 0$: $N_B - m_B g\cos 30° = 0$

$$+\swarrow \Sigma F = ma: \quad m_B g\sin 30° + \mu N_B - 3T = m_B a_B = -\frac{m_B a_A}{3}$$




Eliminate $N_B$.

$$m_B g\left(\sin 30° + \mu\cos 30°\right) - 3T = -\frac{m_B a_A}{3}$$

Eliminate $T$.

$$\left(3m_A g - m_B g\right)\sin 30° - \mu\left(3m_A g + m_B g\right)\cos 30° = \left(3m_A + \frac{m_B}{3}\right)a_A$$

Check the value of $\mu_s$ required for static equilibrium. Set $a_A = 0$ and solve for $\mu$.

$$\mu = \frac{\left(3m_A - m_B\right)\sin 30°}{\left(3m_A + m_B\right)\cos 30°} = \frac{\left(75 - 20\right)}{\left(75 + 20\right)}\tan 30° = 0.334.$$

Since $\mu_s = 0.25 < 0.334,$ sliding occurs.

Calculate $\dfrac{a_A}{g}$ for sliding. Use $\mu = \mu_k = 0.20$.

*continued*


© 2007 The McGraw-Hill Companies.

$$\frac{a_A}{g} = \frac{(3m_A - m_B)\sin 30° - \mu(3m_A + m_B)\cos 30°}{3m_A + m_B/3}$$

$$= \frac{(30 - 8)\sin 30° - (0.20)(30 + 8)\cos 30°}{30 + 2.667} = 0.13525$$

(*a*) $a_A = (0.13525)(9.81) = 1.327 \text{ m/s}^2$ $\quad \mathbf{a}_A = 1.327 \text{ m/s}^2 \measuredangle 30° \blacktriangleleft$

$a_B = -\left(\frac{1}{3}\right)(1.327) = -0.442 \text{ m/s}^2$ $\quad \mathbf{a}_B = 0.442 \text{ m/s}^2 \measuredangle 30° \blacktriangleleft$

(*b*) $T = m_A g(\sin 30° - \mu\cos 30°) - m_A a_A$

$= (10)(9.81)(\sin 30° - 0.20\cos 30°) - (10)(1.327)$

$T = 18.79 \text{ N} \blacktriangleleft$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

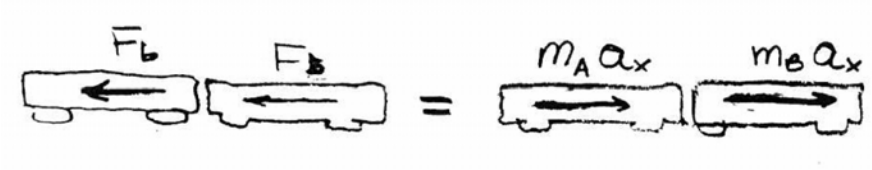
## Chapter 12, Solution 14.

Data: $$m_A = \frac{55000 \text{ lb}}{32.2 \text{ ft/s}^2} = 1708.1 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_B = \frac{44000 \text{ lb}}{32.2 \text{ ft/s}^2} = 1366.5 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$v_0 = -55 \text{ mi/h} = -80.667 \text{ ft/s}$$

(*a*) Use both cars together as a free body. Consider horizontal force components only. Both cars have same acceleration.




$$\stackrel{+}{\rightarrow} \sum F_x = \sum ma_x: \quad -F_b - F_b = m_A a_x + m_B a_x$$

$$a_x = \frac{F_b + F_b}{m_A + m_B} = \frac{7000 + 7000}{1708.1 + 1366.5} = 4.5534 \text{ m/s}^2$$

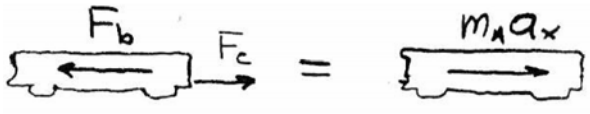
$$a_x = v\frac{dv}{dx}$$

$$\int_0^{x_f} a_x\, dx = \int_{v_0}^{0} v\, dv \qquad a_x x_f = \frac{v_0^2}{2}$$

$$x_f = -\frac{v_0^2}{2a_x} = -\frac{(-80.667)^2}{(2)(4.5534)} = -751 \text{ ft}$$

715 ft to the left ◀

(*b*) Use car *A* as free body. $F_c$ = coupling force.




$$\stackrel{+}{\rightarrow} \sum F_x = \sum ma_x: \quad F_c - F_b = m_A a_x$$

$$F_c = m_A a_x - F_b = (1708.1)(4.5534) + 7000 = 778 \text{ lb}$$

$F_c = 778$ lb tension ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

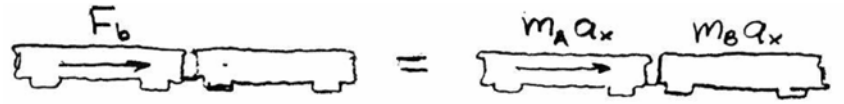
## Chapter 12, Solution 15.

Data: $$m_A = \frac{55000 \text{ lb}}{32.2 \text{ ft/s}^2} = 1708.1 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_B = \frac{44000 \text{ lb}}{32.2 \text{ ft/s}^2} = 1366.5 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$v_0 = -55 \text{ mi/h} = -80.667 \text{ ft/s}$$

(*a*) Use both cars together as a free body. Consider horizontal force components only. Both cars have same acceleration.




$$\stackrel{+}{\rightarrow} \sum F_x = \sum ma_x: \quad -F_b - F_b = m_A a_x + m_B a_x$$

$$a_x = \frac{F_b}{m_A + m_B} = \frac{7000}{1708.1 + 1366.5} = 2.2767 \text{ m/s}^2$$

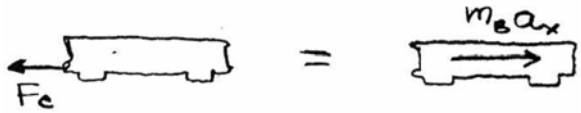
$$a_x = v\frac{dv}{dx}$$

$$\int_0^{x_f} a_x\,dx = \int_{v_0}^{0} v\,dv \qquad a_x x_f = \frac{v_0^2}{2}$$

$$x_f = -\frac{v_0^2}{2a_x} = -\frac{(-80.667)^2}{(2)(2.2767)} = 1429 \text{ ft}$$

1429 ft to the left ◀

(*b*) Use car *B* as a free body. $F_c$ = coupling force.



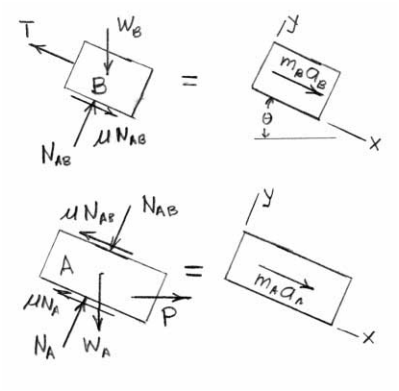

$$\stackrel{+}{\rightarrow} \sum F_x = \sum ma_x: \quad -F_c = m_B a_x$$

$$-F_c = (1366.5)(2.2767) = 3110 \text{ lb}$$

$F_c = 3110$ lb. compression ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 16.




Constraint of cable: $2x_A + (x_B - x_A) = x_A + x_B = \text{constant}.$

$$a_A + a_B = 0, \qquad \text{or} \qquad a_B = -a_A$$

Assume that block $A$ moves down and block $B$ moves up.

Block $B$: $+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B \cos\theta = 0$

$$+\searrow \Sigma F_x = ma: \quad -T + \mu N_{AB} + W_B \sin\theta = \frac{W_B}{g} a_B$$

Eliminate $N_{AB}$ and $a_B$.

$$-T + W_B(\sin\theta + \mu\cos\theta) = W_B \frac{a_B}{g} = -W_B \frac{a_A}{g}$$

Block $A$: $+\nearrow \Sigma F_y = 0$: $N_A - N_{AB} - W_A \cos\theta = 0$

$$N_A = N_{AB} + W_A \cos\theta = (W_B + W_A)\cos\theta$$

$$\Sigma F_x = m_A a_A: \quad -T + W_A \sin\theta - F_{AB} - F_A = \frac{W_A}{g} a_A$$

$$-W_B(\sin\theta + \mu\cos\theta) - W_B \frac{a_A}{g} + W_A \sin\theta - \mu W_B \cos\theta$$

$$-\mu(W_B + W_A)\cos\theta = W_A \frac{a_A}{g}$$

$$(W_A - W_B)\sin\theta - \mu(W_A + 3W_B)\cos\theta = (W_A + W_B)\frac{a_A}{g}$$

Check the condition of impending motion.

$$\mu = \mu_s = 0.20, \qquad a_A = a_B = 0, \qquad \theta = \theta_s$$

$$(W_A - W_B)\sin\theta_s - 0.20(W_A + 3W_B)\cos\theta_s = 0$$

*continued*


© 2007 The McGraw-Hill Companies.

$$\tan\theta_s = \frac{0.20(W_A + 3W_B)}{W_A - W_B} = \frac{(0.20)(128)}{64} = 0.40$$

$\theta_s = 21.8° < \theta = 25°$. The blocks move.

Calculate $\frac{a_A}{g}$ using $\mu = \mu_k = 0.15$ and $\theta = 25°$.

$$\frac{a_A}{g} = \frac{(W_A - W_B)\sin\theta - \mu_k(W_A + 3W_B)\cos\theta}{W_A + W_B}$$

$$= \frac{64\sin 25° - (0.15)(128)\cos 25°}{96} = 0.10048$$

$$a_A = (0.10048)(32.2) = 3.24 \text{ ft/s}^2$$

(*a*) $a_B = -3.24 \text{ ft/s}^2$ $\mathbf{a}_B = 3.24 \text{ ft/s}^2 \angle 25°$ ◀

(*b*) $T = W_B(\sin\theta + \mu\cos\theta) + W_B\frac{a_A}{g}$

$$= 16(\sin 25° + 0.15\cos 25°) + (16)(0.10048)$$

$T = 10.54 \text{ lb}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 17.

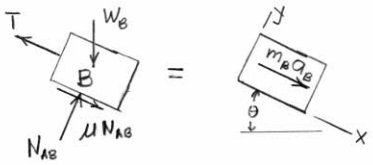
Constraint of cable: $2x_A + (x_B - x_A) = x_A + x_B = \text{constant}.$

$$a_A + a_B = 0, \qquad \text{or} \qquad a_B = -a_A$$

Assume that block $A$ moves down and block $B$ moves up.

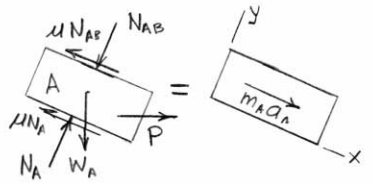
Block $B$: $+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B \cos\theta = 0$

$$+\searrow \Sigma F_x = ma_x: \quad -T + \mu N_{AB} + W_B \sin\theta = \frac{W_B}{g} a_B$$




Eliminate $N_{AB}$ and $a_B$.

$$-T + W_B(\sin\theta + \mu\cos\theta) = W_B \frac{a_B}{g} = -W_B \frac{a_A}{g}$$




Block $A$: $+\nearrow \Sigma F_y = 0$: $N_A - N_{AB} - W_A \cos\theta + P\sin\theta = 0$

$$N_A = N_{AB} + W_A \cos\theta - P\sin\theta$$

$$= (W_B + W_A)\cos\theta - P\sin\theta$$

$$\Sigma F_x = m_A a_A: \quad -T + W_A \sin\theta - F_{AB} - F_A + P\cos\theta = \frac{W_A}{g} a_A$$

$$-W_B(\sin\theta + \mu\cos\theta) - W_B \frac{a_A}{g} + W_A \sin\theta - \mu W_B \cos\theta$$

$$-\mu(W_B + W_A)\cos\theta + \mu P\sin\theta + P\cos\theta = W_A \frac{a_A}{g}$$

$$(W_A - W_B)\sin\theta - \mu(W_A + 3W_B)\cos\theta + P(\mu\sin\theta + \cos\theta) = (W_A + W_B)\frac{a_A}{g}$$

Check the condition of impending motion.

$$\mu = \mu_s = 0.20, \quad a_A = a_B = 0, \quad \theta = 25°$$

$$(W_A - W_B)\sin\theta - \mu_s(W_A + 3W_B)\cos\theta + P_s(\mu_s \sin\theta + \cos\theta) = 0$$

*continued*


© 2007 The McGraw-Hill Companies.

$$P_s = \frac{\mu_s(W_A + 3W_B)\cos\theta - (W_A - W_B)\sin\theta}{\mu_s \sin\theta + \cos\theta}$$

$$= \frac{(0.20)(128)\cos 25° - 64\sin 25°}{0.20\sin 25° + \cos 25°} = -3.88 \text{ lb} < 10 \text{ lb}$$

Blocks will move with $P = 10$ lb.

Calculate $\frac{a_A}{g}$ using $\mu = \mu_k = 0.15$, $\theta = 25°$, and $P = 10$ lb.

$$\frac{a_A}{g} = \frac{(W_A - W_B)\sin\theta - \mu_k(W_A + 3W_B)\cos\theta + P(\mu_k \sin\theta + \cos\theta)}{W_A + W_B}$$

$$= \frac{64\sin 25° - (0.15)(128)\cos 25° + (10)(0.15\sin 25° + \cos 25°)}{96}$$

$$= 0.20149$$

$$a_A = (0.20149)(32.2) = 6.49 \text{ ft/s}^2$$

(*a*) $a_B = -6.49 \text{ ft/s}^2$, $\mathbf{a}_B = 6.49 \text{ ft/s}^2 \measuredangle 25°$ ◀

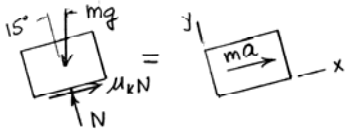
(*b*) $T = W_B(\sin\theta + \mu\cos\theta) + W_B\frac{a_A}{g}$

$$= 16(\sin 25° + 0.15\cos 25°) + (16)(0.20149)$$

$T = 12.16$ lb. ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 18.




Assume $\mathbf{a}_B > \mathbf{a}_A$ so that the boxes separate. Boxes are slipping.

$$\mu = \mu_k$$

$$\Sigma F_y = 0: \quad N - mg\cos 15° = 0$$

$$N = mg\cos 15°$$

$$\Sigma F_x = ma: \quad \mu_k N - mg\sin 15° = ma$$

$$\mu_k mg\cos 15° - mg\sin 15° = ma$$

$$a = g\left(\mu_k \cos 15° - \sin 15°\right), \quad \text{independent of } m.$$

For box $A$, $\mu_k = 0.30$

$$a_A = 9.81\left(0.30\cos 15° - \sin 15°\right) \qquad \text{or} \quad \mathbf{a}_A = 0.304 \text{ m/s}^2 \angle 15° \blacktriangleleft$$

For box $B$, $\mu_k = 0.32$

$$a_B = 9.81\left(0.32\cos 15° - \sin 15°\right) \qquad \text{or} \quad \mathbf{a}_B = 0.493 \text{ m/s}^2 \angle 15° \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 19.

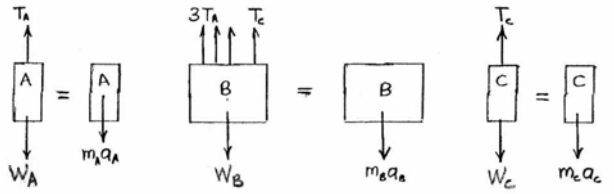
Let $y$ be positive downward position for all blocks.

Constraint of cable attached to mass $A$: $y_A + 3y_B = \text{constant}$

$$a_A + 3a_B = 0 \qquad \text{or} \qquad a_A = -3a_B$$

Constraint of cable attached to mass $C$: $y_C + y_B = \text{constant}$

$$a_C + a_B = 0 \qquad \text{or} \qquad a_C = -a_B$$




For each block $+\downarrow \Sigma F = ma$:

Block $A$: $W_A - T_A = m_A a_A$, or $T_A = W_A - m_A a_A = W_A - 3m_A a_B$

Block $C$: $W_C - T_C = m_C a_C$, or $T_C = W_C - m_C a_C = W_C - m_C a_B$

Block $B$: $W_B - 3T_A - T_C = m_B a_B$

$$W_B - 3\left(W_A - 3m_A a_B\right) - \left(W_C - m_C a_B\right) = m_B a_B$$

$$\text{or} \quad \frac{a_B}{g} = \frac{W_B - 3W_A - W_C}{W_B + 9W_A + W_C} = \frac{60 - 60 - 20}{60 + 180 + 20} = -0.076923$$

(*a*) Accelerations. $a_B = \left(-0.076923\right)\left(32.2\right) = -2.477 \text{ ft/s}^2$ $\quad \mathbf{a}_B = 2.48 \text{ ft/s}^2 \uparrow$ ◀

$a_A = -\left(3\right)\left(-2.477\right) = 7.431 \text{ ft/s}^2$ $\quad \mathbf{a}_A = 7.43 \text{ ft/s}^2 \downarrow$ ◀

$a_C = -\left(-2.477\right) = 2.477 \text{ ft/s}^2$ $\quad \mathbf{a}_C = 2.48 \text{ ft/s}^2 \downarrow$ ◀

(*b*) Tensions. $T_A = 20 - \dfrac{20}{32.2}\left(7.43\right)$ $\quad T_A = 15.38 \text{ lb}$ ◀

$T_C = 20 - \dfrac{20}{32.2}\left(2.477\right)$ $\quad T_C = 18.46 \text{ lb}$ ◀

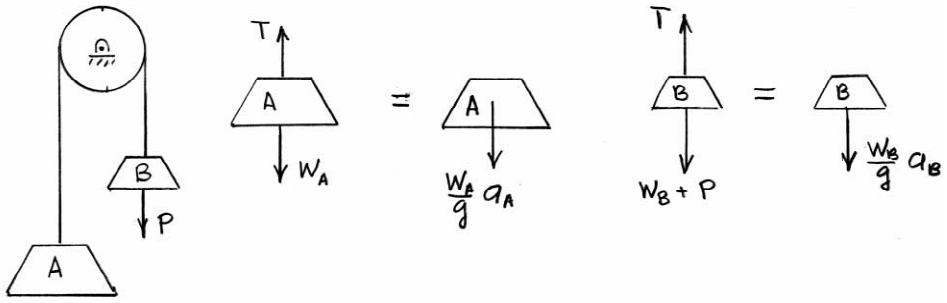
*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 20.

Let *y* be positive downward for both blocks.

Constraint of cable: $y_A + y_B = \text{constant}$

$$a_A + a_B = 0 \qquad \text{or} \qquad a_B = -a_A$$




For blocks *A* and *B*, $+\downarrow \Sigma F = ma$:

$$\text{Block } A: \quad W_A - T = \frac{W_A}{g} a_A \qquad \text{or} \qquad T = W_A - \frac{W_A}{g} a_A$$

$$\text{Block } B: \quad P + W_B - T = \frac{W_B}{g} a_B = -\frac{W_B}{g} a_A$$

$$P + W_B - W_A + \frac{W_A}{g} a_A = -\frac{W_B}{g} a_A$$

Solving for $a_A$, $\quad a_A = \dfrac{W_A - W_B - P}{W_A + W_B} g$ (1)

$$v_A^2 - (v_A)_0^2 = 2a_A\left[y_A - (y_A)_0\right] \quad \text{with} \quad (v_A)_0 = 0$$

$$v_A = \sqrt{2a_A\left[y_A - (y_A)_0\right]} \tag{2}$$

$$v_A - (v_A)_0 = a_A t \quad \text{with} \quad (v_A)_0 = 0$$

$$t = \frac{v_A}{a_A} \tag{3}$$

*continued*


© 2007 The McGraw-Hill Companies.

(*a*) Acceleration of block *A*.

System (1): $W_A = 100$ lb, $W_B = 50$ lb, $P = 0$

By formula (1), $(a_A)_1 = \dfrac{100 - 50}{100 + 50}(32.2)$ $(\mathbf{a}_A)_1 = 10.73$ ft/s$^2 \downarrow$ ◀

System (2): $W_A = 100$ lb, $W_B = 0$, $P = 50$ lb

By formula (1), $(a_A)_2 = \dfrac{100 - 50}{100}(32.2)$ $(\mathbf{a}_A)_2 = 16.10$ ft/s$^2 \downarrow$ ◀

System (3): $W_A = 1100$ lb, $W_B = 1050$ lb, $P = 0$

By formula (1), $(a_A)_3 = \dfrac{1100 - 1050}{1100 + 1050}(32.2)$ $(\mathbf{a}_A)_3 = 0.749$ ft/s$^2 \downarrow$ ◀

(*b*) $v_A$ at $y_A - (y_A)_0 = 5$ ft. Use formula (2).

System (1): $(v_A)_1 = \sqrt{(2)(10.73)(5)}$ $(v_A)_1 = 10.36$ ft/s $\downarrow$ ◀

System (2): $(v_A)_2 = \sqrt{(2)(16.10)(5)}$ $(v_A)_2 = 12.69$ ft/s $\downarrow$ ◀

System (3): $(v_A)_3 = \sqrt{(2)(0.749)(5)}$ $(v_A)_3 = 2.74$ ft/s $\downarrow$ ◀

(*c*) Time at $v_A = 10$ ft/s. Use formula (3).

System (1): $t_1 = \dfrac{10}{10.73}$ $t_1 = 0.932$ s ◀

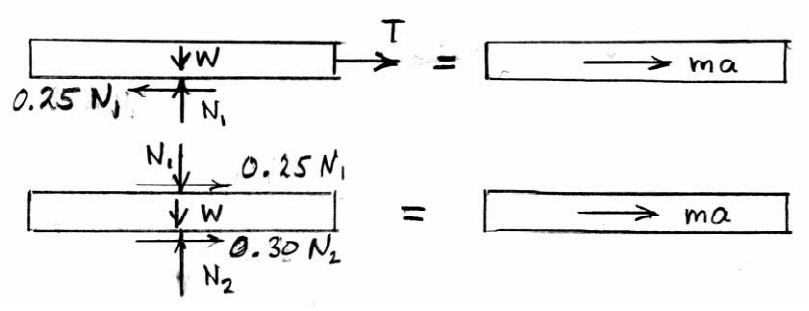
System (2): $t_2 = \dfrac{10}{16.10}$ $t_2 = 0.621$ s ◀

System (3): $t_3 = \dfrac{10}{0.749}$ $t_3 = 13.35$ s ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 21.

(*a*) Maximum acceleration. The cable secures the upper beam; only the lower beam can move.




For the upper beam, $\Sigma F_y = 0$: $N_1 - W = 0$

$$N_1 = W = mg$$

For the lower beam, $\Sigma F_y = 0$: $N_2 - N_1 - W = 0$ or $N_2 = 2W$

$$\pm\rightarrow \Sigma F_x = ma\text{:}\quad 0.25\,N_1 + 0.30\,N_2 = (0.25 + 0.60)W = ma$$

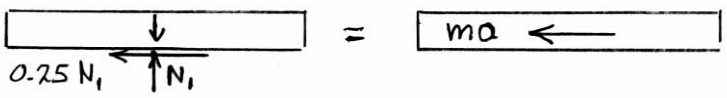
$$a = 0.85\,\frac{W}{m} = (0.85)(32.2) = 23.37 \text{ ft/s}^2 \qquad \mathbf{a} = 27.4 \text{ ft/s}^2 \rightarrow \blacktriangleleft$$

For the upper beam, $\pm\rightarrow \Sigma F_x = ma$: $T - 0.25\,N_1 = ma$

$$T = 0.25W + ma = (0.25)(3000) + \left(\frac{3000}{32.2}\right)(23.37) = 2927 \text{ lb} \qquad T = 2930 \text{ lb} \blacktriangleleft$$

(*b*) Maximum deceleration of trailer.

Case 1: Assume that only the top beam slips. As in Part (*a*) $N_1 = mg$.



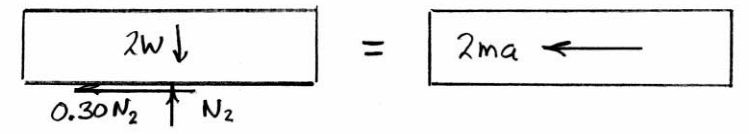

$$\pm\leftarrow \Sigma F = ma\text{:}\quad 0.25W = ma$$

$$a = 0.25g = 8.05 \text{ ft/s}^2$$

*continued*

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

Case 2: Assume that both beams slip. As before $N_2 = 2W$.



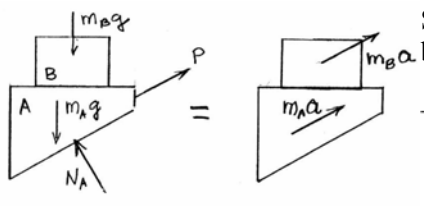

$$\overset{+}{\leftarrow} \Sigma F = (2m)a: \quad (0.30)(2W) = (2m)a$$

$$a = 0.30g = 9.66 \text{ ft/s}^2$$

The smaller deceleration value governs. $a = 8.05 \text{ ft/s}^2$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 22.




Since both blocks move together, they have a common acceleration. Use blocks $A$ and $B$ together as a free body.

$+\nearrow \Sigma F = \Sigma\, ma$:

$$P - m_A g \sin 30° - m_B g \sin 30° = (m_A + m_B)a$$

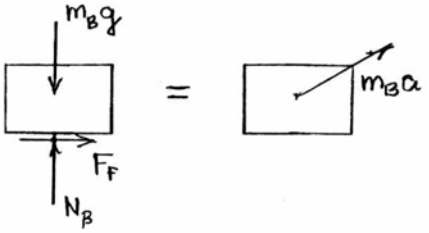
$$a = \frac{P}{m_A + m_B} - g \sin 30° = \frac{500}{50} - 9.81 \sin 30°$$

$$= 5.095 \text{ m/s}^2$$

Use block $B$ as a free body.

$\xrightarrow{+} \Sigma F = m_B a \cos 30°$: $\quad F_f = m_B a \cos 30°$

$$F_f = (10)(5.095)\cos 30° = 44.124 \text{ N}$$




$+\uparrow \Sigma F = m_B a \sin 30°$: $\quad N_B - m_B g = m_B a \sin 30°$

$$N_B = m_B(g + a \sin 30°) = 10(9.81 + 5.095 \sin 30°)$$

$$= 123.575 \text{ N}$$

Minimum coefficient of static friction:

$$\mu_{\min} = \frac{F_f}{N_B} = \frac{44.124}{123.575} \qquad \mu_{\min} = 0.357 \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 23.

(*a*) Kinematics of the belt. $v_o = 0$

1. Acceleration phase with $a_1 = 3.2 \text{ m/s}^2 \rightarrow$

$$v_1 = v_o + a_1 t_1 = 0 + (3.2)(1.5) = 4.8 \text{ m/s}$$

$$x_1 = x_o + v_o t_1 + \frac{1}{2} a_1 t_1^2 = 0 + 0 + \frac{1}{2}(3.2)(1.5)^2 = 3.6 \text{ m}$$

2. Deceleration phase. $v_2 = 0$ since the belt stops.

$$v_2^2 - v_1^2 = 2a_2(x_2 - x_1)$$

$$a_2 = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{0 - (4.8)^2}{2(4.6 - 3.6)} = -11.52 \qquad \mathbf{a}_2 = 11.52 \text{ m/s}^2 \leftarrow$$

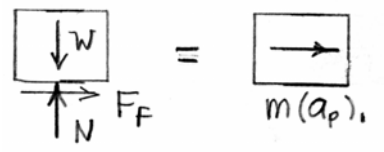
$$t_2 - t_1 = \frac{v_2 - v_1}{a_2} = \frac{0 - 4.8}{-11.52} = 0.41667 \text{ s}$$

(*b*) Motion of the package.

1. Acceleration phase. Assume no slip. $(a_p)_1 = 3.2 \text{ m/s}^2 \rightarrow$

$$\Sigma F_y = 0: \quad N - W = 0 \text{ or } N = W = mg$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma: \quad F_f = m(a_p)_1$$

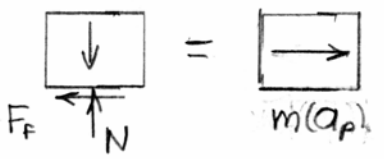



The required friction force is $F_f$.

The available friction force is $\mu_s N = 0.35\, W = 0.35\, mg$

$$\frac{F_f}{m} = (a_p)_1 < \frac{\mu_s N}{m} = \mu_s g = (0.35)(9.81) = 3.43 \text{ m/s}^2$$


© 2007 The McGraw-Hill Companies.




Since $3.2 \text{ m/s}^2 < 3.43 \text{ m/s}^2$, the package does not slip.

$$\left(v_p\right)_1 = v_1 = 4.8 \text{ m/s} \text{ and } \left(x_p\right)_1 = 3.6 \text{ m.}$$

2. Deceleration phase. Assume no slip. $\left(a_p\right)_2 = -11.52 \text{ m/s}^2$

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma: \quad -F_f = m\left(a_p\right)_2$$

$$\frac{F_f}{m} = \left(a_p\right)_2 = -11.52 \text{ m/s}^2$$

$$\frac{\mu_s N}{m} = \frac{\mu_s mg}{m} = \mu_s g = 3.43 \text{ m/s}^2 < 11.52 \text{ m/s}^2$$

Since the available friction force $\mu_s N$ is less than the required friction force $F_f$ for no slip, the package does slip.

$$\left(a_p\right)_2 < 11.52 \text{ m/s}^2, \quad F_f = \mu_k N$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = m\left(a_p\right)_2: \quad -\mu_k N = m\left(a_p\right)_2$$

$$\left(a_p\right)_2 = -\frac{\mu_k N}{m} = -\mu_k g = -(0.25)(9.81) = -2.4525 \text{ m/s}^2$$

$$\left(v_p\right)_2 = \left(v_p\right)_1 + \left(a_p\right)_2 (t_2 - t_1) = 4.8 + (-2.4525)(0.41667) = 3.78 \text{ m/s}^2$$

$$\left(x_p\right)_2 = \left(x_p\right)_1 + \left(v_p\right)_1 (t_2 - t_1)^2 + \frac{1}{2}\left(a_p\right)_2 (t_2 - t_1)^2$$

$$= 3.6 + (4.8)(0.41667) + \frac{1}{2}(-2.4525)(0.41667)^2 = 5.387 \text{ m}$$

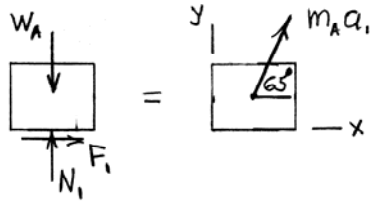
Position of package relative to the belt

$$\left(x_p\right)_2 - x_2 = 5.387 - 4.6 = 0.787 \qquad x_{p/\text{belt}} = 0.787 \text{ m} \rightarrow$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 24.**

Acceleration $\mathbf{a}_1$: Impending slip. $F_1 = \mu_s N_1 = 0.30\ \mathrm{N}_1$




$$\Sigma F_y = m_A a_y: \quad N_1 - W_A = m_A a_1 \sin 65°$$

$$N_1 = W_A + m_A a_1 \sin 65°$$

$$= m_A\left(g + a_1 \sin 65°\right)$$

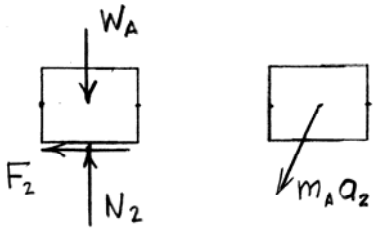
$$\pm \Sigma F_x = m_A a_x: \quad F_1 = m_A a_1 \cos 65°$$

$$F_1 = \mu_s N \quad \text{or} \quad m_A a_1 \cos 65° = 0.30 m_A\left(g + a_1 \sin 65°\right)$$

$$a_1 = \frac{0.30\ g}{\cos 65° - 0.30 \sin 65°} = (1.990)(9.81) = 19.53\ \mathrm{m/s^2}$$

$$\mathbf{a}_1 = 19.53\ \mathrm{m/s^2} \angle 65° \blacktriangleleft$$

Deceleration $\mathbf{a}_2$: Impending slip. $F_2 = \mu_S N_2 = 0.30\ \mathrm{N}_2$




$$\Sigma F_y = ma_y: \quad N_1 - W_A = -m_A a_2 \sin 65°$$

$$N_1 = W_A - m_A a_2 \sin 65°$$

$$\pm \Sigma F_x = ma_x: \quad F_2 = m_A a_2 \cos 65°$$

$$F_2 = \mu_S N_2 \qquad \text{or} \qquad m_A a_2 \cos 65° = 0.30 m_A\left(g - a_2 \cos 65°\right)$$

$$a_2 = \frac{0.30\ g}{\cos 65° + 0.30 \sin 65°} = (0.432)(9.81) = 4.24\ \mathrm{m/s^2}$$

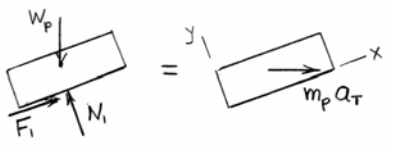
$$\mathbf{a}_2 = 4.24\ \mathrm{m/s^2} \angle 65° \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 25.

Let $\mathbf{a}_P$ be the acceleration of the plywood, $\mathbf{a}_T$ be the acceleration of the truck, and $\mathbf{a}_{P/T}$ be the acceleration of the plywood relative to the truck.

(*a*) Find the value of $\mathbf{a}_T$ so that the relative motion of the plywood with respect to the truck is impending. $a_P = a_T$ and $F_1 = \mu_s N_1 = 0.40\ \mathrm{N}_1$




$$+\nwarrow \Sigma F_y = m_P a_y: \quad N_1 - W_P \cos 20^\circ = -m_P a_T \sin 20^\circ$$

$$N_1 = m_P\left(g\cos 20^\circ - a_T \sin 20^\circ\right)$$

$$+\nearrow \Sigma F_x = ma_x: \quad F_1 - W_P \sin 20^\circ = m_P a_T \cos 20^\circ$$

$$F_1 = m_P\left(g\sin 20^\circ + a_T \cos 20^\circ\right)$$

$$m_P\left(g\sin 20^\circ + a_T\cos 20^\circ\right) = 0.40\, m_P\left(g\cos 20^\circ - a_T \sin 20^\circ\right)$$

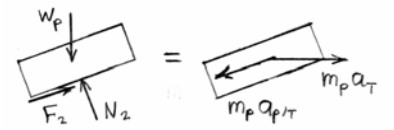
$$a_T = \frac{\left(0.40\cos 20^\circ - \sin 20^\circ\right)}{\cos 20^\circ + 0.40\sin 20^\circ} g = \left(0.03145\right)\left(9.81\right) = 0.309$$

$$\mathbf{a}_T = 0.309\ \mathrm{m/s^2} \rightarrow \quad \blacktriangleleft$$

(*b*) $$x_{P/T} = \left(x_{P/T}\right)_o + \left(v_{P/T}\right)t + \frac{1}{2}a_{P/T}t^2 = 0 + 0 + \frac{1}{2}a_{P/T}t^2$$

$$a_{P/T} = \frac{2x_{P/T}}{t^2} = \frac{(2)(1)}{(0.4)^2} = 12.5\ \mathrm{m/s^2}, \qquad \mathbf{a}_{P/T} = 12.5\ \mathrm{m/s^2} \measuredangle 20^\circ$$

$$\mathbf{a}_P = \mathbf{a}_T + \mathbf{a}_{P/T} = \left(a_T \rightarrow\right) + \left(12.5\ \mathrm{m/s^2} \measuredangle 20^\circ\right)$$




$$+\nwarrow F_y = m_P a_y:$$

$$N_2 - W_P \cos 20^\circ = -m_P a_T \sin 20^\circ$$

$$N_2 = m_P\left(g\cos 20^\circ - a_T \sin 20^\circ\right)$$

*continued*


© 2007 The McGraw-Hill Companies.

$$+\nearrow \Sigma F_x = \Sigma m a_x:$$

$$F_2 - W_P \sin 20^\circ = m_P a_T \cos 20^\circ - m_P a_{P/T}$$

$$F_2 = m_P\left(g \sin 20^\circ + a_T \cos 20^\circ - a_{P/T}\right)$$

For sliding with friction $F_2 = \mu_k N_2 = 0.30\ N_2$

$$m_P\left(g \sin 20^\circ + a_T \cos 20^\circ - a_{P/T}\right) = 0.30 m_P\left(g \cos 20^\circ + a_T \sin 20^\circ\right)$$

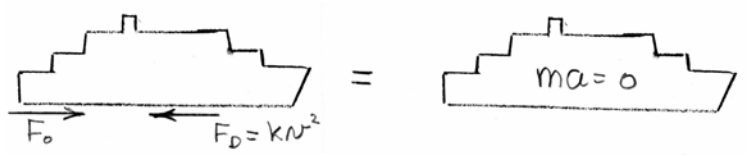
$$a_T = \frac{\left(0.30 \cos 20^\circ - \sin 20^\circ\right) g + a_{P/T}}{\cos 20^\circ + 0.30 \sin 20^\circ}$$

$$= \left(-0.05767\right)\left(9.81\right) + \left(0.9594\right)\left(12.5\right) = 11.43$$

$$\mathbf{a}_T = 11.43\ \text{m/s}^2 \rightarrow \blacktriangleleft$$

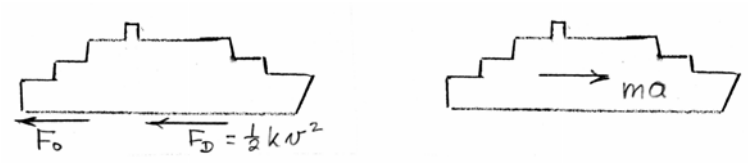
*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 26.




At maximum speed $a = 0$. $\quad F_0 = kv_0^2 = 0 \quad k = \dfrac{F_0}{v_0^2}$

When the propellers are reversed, $F_0$ is reversed.




$$\pm \Sigma F_x = ma: \quad -F_0 - kv^2 = ma$$

$$-F_0 - F_0\frac{v^2}{v_0^2} = ma \qquad a - \frac{F_0}{mv_0^2}\left(v_0^2 + v^2\right)$$

$$dx = \frac{vdv}{a} = \frac{mv_0^2 vdv}{F_0\left(v_0^2 + v^2\right)}$$

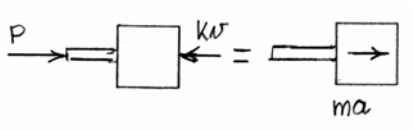
$$\int_0^x dx = -\frac{mv_0^2}{F_0}\int_{v_0}^0 \frac{vdv}{v_0^2 + v^2}$$

$$x = -\frac{mv_0^2}{F_0}\ \frac{1}{2}\ \ln\left(v_0^2 + v^2\right)\Big|_{v_0}^0 = -\frac{mv_0^2}{2F_0}\left[\ln v_0^2 - \ln\left(2v_0^2\right)\right] = \frac{mv_0^2}{2F_0}\ln 2$$

$$x = 0.347\frac{m_0 v_0^2}{F_0} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 27.




$$\stackrel{+}{\rightarrow}\Sigma F = ma: \quad P - kv = ma$$

$$\frac{dv}{dt} = a = \frac{P - kv}{m}$$

$$\int_0^t dt = \int_0^v \frac{m\,dv}{P - kv} = -\frac{m}{k}\ln\left(P - kv\right)\Big|_0^v = -\frac{m}{k}\left[\ln\left(P - kv\right) - \ln P\right]$$

$$t = -\frac{m}{k}\ln\frac{P - kv}{P} \qquad \text{or} \qquad \ln\frac{P - kv}{m} = -\frac{kt}{m}$$

$$\frac{P - kv}{m} = e^{-kt/m} \qquad \text{or} \qquad v = \frac{P}{k}\left(1 - e^{-kt/m}\right)$$

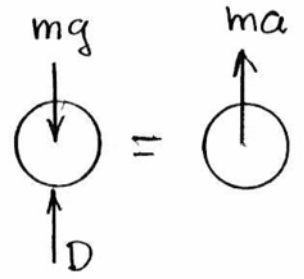
$$x = \int_0^t v\,dt = \frac{Pt}{k}\Big|_0^t - \frac{P}{k}\left(-\frac{k}{m}e^{-kt/m}\right)\Big|_0^t$$

$$= \frac{Pt}{k} + \frac{P}{m}\left(e^{-kt/m} - 1\right) = \frac{Pt}{k} - \frac{P}{m}\left(1 - e^{-kt/m}\right)$$

$$x = \frac{Pt}{k} - \frac{kv}{m}, \text{ which is linear.}$$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 28.




Let $y$ be the position coordinate of the projectile measured upward from the ground. The velocity and acceleration $a$ taken to positive upward. $D = kv^2$.

($a$) Upward motion. $+\uparrow \Sigma F_y = ma$:

$$-D - mg = ma$$

$$a = -\left(g + \frac{D}{m}\right) = -\left(g + \frac{kv^2}{m}\right)$$

$$v\frac{dv}{dy} = -\left(g + \frac{kv^2}{m}\right) = -\frac{k}{m}\left(v^2 + \frac{mg}{k}\right)$$

$$\frac{v\,dv}{v^2 + \frac{mg}{k}} = -\frac{k}{m}dy$$

$$\int_{v_0}^{0} \frac{v\,dv}{v^2 + \frac{mg}{k}} = -\frac{k}{m}\int_0^h dy$$

$$\frac{1}{2}\ln\left(v^2 + \frac{mg}{k}\right)\Bigg|_{v_0}^{0} = -\frac{kh}{m}$$

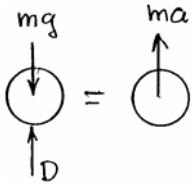
$$\frac{1}{2}\ln\frac{\frac{mg}{k}}{v_0^2 + \frac{mg}{k}} = -\frac{1}{2}\ln\left(\frac{kv_0^2}{mg} + 1\right) = -\frac{kh}{m}$$

$$h = \frac{m}{2k}\ln\left(\frac{kv_0^2}{mg} + 1\right) = \frac{4}{(2)(0.0024)}\ln\left[\frac{(0.0024)(90)^2}{(4)(9.81)} + 1\right]$$

$$= 335.36 \text{ m} \qquad h = 335 \text{ m} \blacktriangleleft$$

*continued*


© 2007 The McGraw-Hill Companies.




(*b*) Downward motion. $+\uparrow \Sigma F = ma$:

$$D - mg = ma$$

$$a = \frac{D}{m} - g = \frac{kv^2}{m} - g$$

$$v\frac{dv}{dy} = \frac{kv^2}{m} - g = -\frac{k}{m}\left(\frac{mg}{k} - v^2\right)$$

$$\frac{v\,dv}{\frac{mg}{k} - v^2} = -\frac{k}{m}dy$$

$$\int_0^{v_f} \frac{v\,dv}{mg} = -\frac{k}{m}\int_h^0 dy$$

$$\frac{1}{2}\ln\left(\frac{mg}{k} - v^2\right)\Big|_0^{v_f} = \frac{kh}{m}$$

$$-\frac{1}{2}\ln\left(\frac{\frac{mg}{k} - v_f^2}{\frac{mg}{k}}\right) = \frac{kh}{m}$$

$$\ln\left(1 - \frac{kv_f^2}{mg}\right) = -\frac{2\,kh}{m}$$

$$1 - \frac{kv_f^2}{mg} = e^{-2\,kh/m}$$

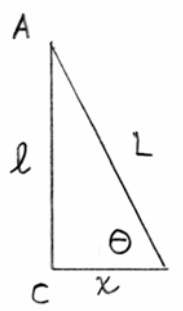
$$v_f = \pm\sqrt{\frac{mg}{k}\left(1 - e^{-2\,kh/m}\right)}$$

$$v_f = \sqrt{\frac{(4)(9.81)}{0.0024}\left[1 - e^{-(2)(0.0024)(335.36)/4}\right]}$$

$$= 73.6 \text{ m/s} \qquad v_f = 73.6 \text{ m/s} \downarrow \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 29.



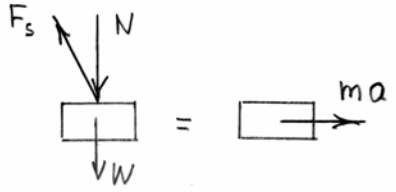

Choose the origin at point $C$, and let $x$ be positive to the right. Then $x$ is a position coordinate of the slider $B$, and $x_0$ is its initial value. Let $L$ be the stretched length of the spring. Then, from the right triangle

$$L = \sqrt{\ell^2 + x^2}$$

The elongation of the spring is $e = L - \ell$, and the magnitude of the force exerted by the spring is

$$F_s = ke = k\left(\sqrt{\ell^2 + x^2} - \ell\right)$$

By geometry, $\cos\theta = \dfrac{x}{\sqrt{\ell^2 + x^2}}$




$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F_s\cos\theta = ma$$

$$-k\left(\sqrt{\ell^2 + x^2} - \ell\right)\frac{x}{\sqrt{\ell^2 + x^2}} = ma$$

$$a = -\frac{k}{m}\left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)$$

$$\int_0^v v\,dv = \int_{x_0}^0 a\,dx$$

$$\frac{1}{2}v^2\Big|_0^v = -\frac{k}{m}\int_{x_0}^0\left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)dx = -\frac{k}{m}\left(\frac{1}{2}x^2 - \ell\sqrt{\ell^2 + x^2}\right)\Big|_{x_0}^0$$

$$\frac{1}{2}v^2 = -\frac{k}{m}\left(0 - \ell^2 - \frac{1}{2}x_0^2 + \ell\sqrt{\ell^2 + x_0^2}\right)$$

$$v^2 = \frac{k}{m}\left(2\ell^2 + x_0^2 - 2\ell\sqrt{\ell^2 + x_0^2}\right)$$

$$= \frac{k}{m}\left[\left(\ell^2 + x_0^2\right) - 2\ell\sqrt{\ell^2 + x_0^2} + \ell^2\right]$$

answer: $v = \sqrt{\dfrac{k}{m}}\left(\sqrt{\ell^2 + x_0^2} - \ell\right)$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 30.

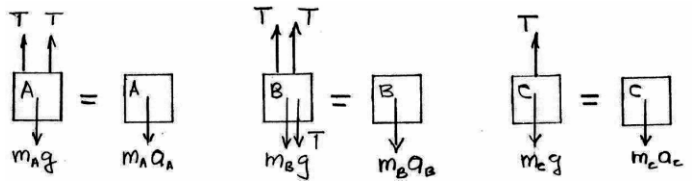
Let $y_A$, $y_B$, and $y_C$ be the position coordinates of blocks *A*, *B*, and *C* respectively measured downward from the upper support. Then the corresponding velocities and accelerations are positive downward.

Constraint of cable: $y_A - y_B + y_A + 2y_B + y_C = \text{constant}$

Differentiating twice, $a_A - a_B + a_A + 2a_B + a_C = 0$

$$2a_A + a_B + a_C = 0 \tag{1}$$

Draw free body diagrams of each of the blocks.




Block *A*. $+\downarrow \Sigma F = ma$: $m_A g - 2T = m_A a_A$

$$a_A = g - \frac{2T}{m_A} \tag{2}$$

Block *B*. $+\downarrow \Sigma F = ma$: $m_B g - T = m_B a_B$

$$a_B = g - \frac{T}{m_B} \tag{3}$$

Block *C*. $+\downarrow \Sigma F = ma$: $m_C g - T = m_C a_C$

$$a_C = g - \frac{T}{m_C} \tag{4}$$

Substitute (2), (3) and (4) into (1).

$$2\left(g - \frac{2T}{m_A}\right) + \left(g - \frac{T}{m_B}\right) + \left(g - \frac{T}{m_C}\right) = 0$$

$$4g - \left(\frac{4}{m_A} + \frac{1}{m_B} + \frac{1}{m_C}\right)T = 0$$

$$(4)(9.81) - \left(\frac{4}{10} + \frac{1}{10} + \frac{1}{10}\right)T = 0 \qquad T = 65.4 \text{ N}$$

Substitute into (2), $a_A = 9.81 - \dfrac{(2)(65.4)}{10} = -3.27 \text{ m/s}^2$

(*a*) Change in position. $y_A - (y_A)_0 = \dfrac{1}{2} a_A t^2$

$$y_A - (y_A)_0 = \frac{1}{2}(-3.27)(0.5)^2 = -0.409 \text{ m}$$

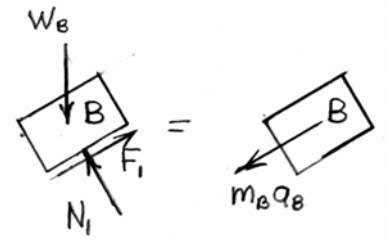
$\Delta y = 0.409 \text{ m} \uparrow$ ◀

(*b*) Tension in the cable. $T = 65.4 \text{ N}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 31.

$W_A = m_A g = 98.1$ N $\qquad W_B = m_B g = 49.05$ N




Assume that block $B$ slides downward relative to block $A$. Then the friction force $F_1$ is directed as shown. Its magnitude is

$$F_1 = \mu_k N_1 = 0.10\ \mathrm{N}_1.$$

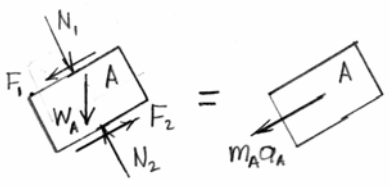
$$+\nwarrow \Sigma F_y = 0: N_1 - W_B \cos 30° = 0,\ N_1 = W_B \cos 30° = 49.05 \cos 30° = 42.48 \text{ N}.$$

$$F_1 = (0.10)(42.48) = 4.248 \text{ N}.$$

$$+\swarrow \Sigma F_x = m_B a_B: \quad W_B \sin 30° - F_1 = m_B a_B$$

$$a_B = \frac{1}{m_B}(W_B \sin 30° - F_1) = \frac{1}{5}(49.05 \sin 30° - 4.248) = 4.055 \text{ m/s}^2$$

Assume that block $A$ slides downward relative to the fixed plane. The friction force $F_2$ is directed as shown. Its magnitude is




$$F_2 = \mu_k N_2 = 0.20\ \mathrm{N}_2.$$

$$+\nwarrow F_y = 0: N_2 - N_1 - W_A \cos 30° = 0,\ N_2 = 42.48 + 98.1 \cos 30° = 127.44 \text{ N}.$$

$$F_2 = (0.20)(127.44) = 25.49 \text{ N}$$

$$+\swarrow \Sigma F_x = m_A a_A: \quad W_A \sin 30° - F_2 + F_1 = m_A a_A$$

$$a_A = \frac{1}{m_A}(W_A \sin 30° - F_2 + F_1)$$

$$= \frac{1}{10}(98.1 \sin 30° - 25.49 + 4.248) = 2.781 \text{ m/s}^2$$

$$a_{B/A} = a_B - a_A = 4.055 - 2.781 = 1.274 \text{ m/s}^2$$

Since both $a_{B/A}$ and $a_A$ are positive, the directions of relative motion are as assumed above.

(*a*) Acceleration of block $A$. $\qquad \mathbf{a}_A = 2.78 \text{ m/s}^2 \measuredangle 30°$ ◀

(*b*) Velocity of $B$ relative to $A$ at $t = 0.5$ s.

$$v_{B/A} = a_{B/A} t = (1.274)(0.5) \qquad \mathbf{v}_{B/A} = 0.637 \text{ m/s} \measuredangle 30°$$ ◀

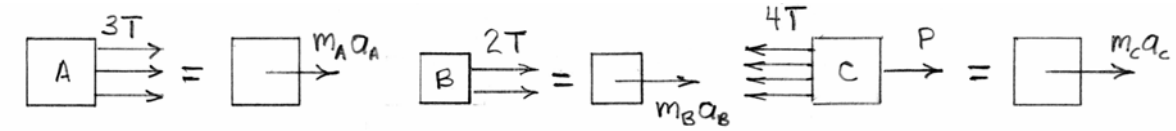

© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 32.

Let the positive direction for position coordinates, velocities, and accelerations be to the right. Let the origin lie at the fixed anchor.

Constraint of cable: $3(x_C - x_A) + (x_C - x_B) + (-x_B) = \text{constant}$

$$4a_C - 2a_B - 3a_A = 0 \tag{1}$$




$\pm \Sigma F_x = ma_x$:

Block $A$: $3T = m_A a_A$ or $a_A = \dfrac{3T}{m_A} = \dfrac{3T}{20}\, g$ (2)

Block $B$: $2T = m_B a_B$ or $a_B = \dfrac{2T}{m_B} = \dfrac{2T}{10}\, g$ (3)

Block $C$: $P - 4T = m_C a_C$ or $a_C = \dfrac{P - 4T}{20} = \dfrac{P - 4T}{20}\, g$ (4)

Substituting (2), (3), and (4) into (1),

$$4\left(\frac{P - 4T}{20}\right) - 2\left(\frac{2T}{10}\right) - 3\left(\frac{3T}{20}\right) = 0$$

$$\left(\frac{16}{20} + \frac{4}{10} + \frac{9}{20}\right)T = \frac{4P}{20}$$

$$T = (0.12121)P = (0.12121)(50) = 6.0605 \text{ lb}$$

(*a*) From (2), $a_A = \dfrac{(3)(6.0605)(32.2)}{20} = 29.3 \text{ ft/s}^2$ $\mathbf{a}_A = 29.3 \text{ ft/s}^2 \rightarrow$ ◀

From (3), $a_B = \dfrac{(2)(6.0605)(32.2)}{10} = 39.0 \text{ ft/s}^2$ $\mathbf{a}_B = 39.0 \text{ ft/s}^2 \rightarrow$ ◀

From (4), $a_C = \dfrac{\left[50 - (4)(6.0605)\right](32.2)}{20} = 41.5 \text{ ft/s}^2$ $\mathbf{a}_C = 41.5 \text{ m/s}^2 \rightarrow$ ◀

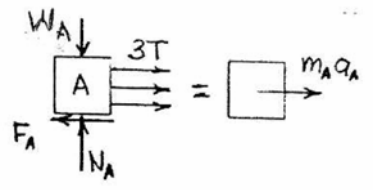
(*b*) As determined above, $T = 6.06 \text{ lb}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 33.

$$m_A = m_C = \frac{20}{32.2} = 0.62112 \text{ lb}\cdot\text{s}^2/\text{ft}, \quad m_B = \frac{10}{32.2} = 0.31056 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Let the positive direction for position coordinates, velocities, and accelerations be to the right. Let the origin lie at the fixed anchor.




Constraint of cable: $3(x_C - x_A) + (x_C - x_B) + (-x_B) = \text{constant}$

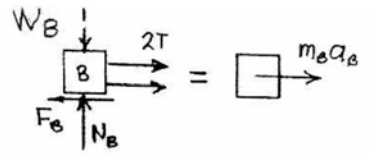
$$4a_C - 2a_B - 3a_A = 0 \tag{1}$$

Block $A$: $+\uparrow \Sigma F_y = 0$: $N_A - W_A = 0$

$$N_A = W_A, \; F_A = \mu_k N_A = \mu_\kappa W_A$$

$$+\rightarrow \Sigma F_x = m_A a_x: \quad 3T - F_A = m_A a_A$$

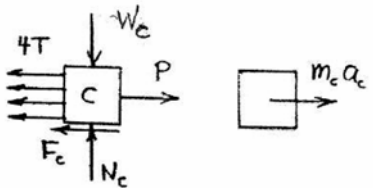
$$a_A = \frac{3T - \mu_k W_A}{m_A} = \frac{3T}{m_A} - 0.20g \tag{2}$$




Block $B$: $N_B = W_B, \quad F_B = \mu_k W_B$

$$+\rightarrow \Sigma F_x = m_B a_B: \quad 2T - F_B = m_B a_B$$

$$a_B = \frac{2T - \mu_k W_B}{m_B} = \frac{2T}{m_B} - 0.20g \tag{3}$$




Block $C$: $N_C = W_C, \; F_C = \mu_k W_C$

$$+\rightarrow \Sigma F_x = m_C a_A: \quad P - 4T - F_C = m_C a_C \tag{4}$$

Kinematics: $x_C = (x_C)_0 + (v_C)_0 + \frac{1}{2} a_C t^2 = 0 + \frac{1}{2} a_C t^2$

$$a_C = \frac{2\left[x_C - (x_C)_0\right]}{t^2} = \frac{(2)(2.4)}{(0.4)^2} = 30 \text{ ft/s}^2 \tag{5}$$


© 2007 The McGraw-Hill Companies.

Substitute (2), (3) and (5) into (1).

$$(4)(30) - (2)\left(\frac{2T}{0.31056} - 0.20g\right) - (3)\left(\frac{3T}{0.62112} - 0.20g\right)$$

$$= (4)(30) - (2)\left[\frac{2T}{0.31056} - (0.2)(32.2)\right] - (3)\left[\frac{3T}{0.62112} - (0.2)(32.2)\right]$$

$$= 120 - 27.37\,T + 32.2 = 0 \qquad T = 5.5608 \text{ lb}$$

From (4), $P = 4T + F_C + m_C a_C$

$$= (4)(5.5608) + (0.20)(20) + (0.62112)(30)$$

$$= 44.877 \text{ lb}$$

From (2), $a_A = \dfrac{(3)(5.5608)}{0.62112} - (0.20)(32.2) = 20.42 \text{ ft/s}^2$

From (3), $a_B = \dfrac{(2)(5.5608)}{0.31056} - (0.20)(32.2) = 29.37 \text{ ft/s}^2$

(*a*) Acceleration vectors. $\mathbf{a}_A = 20.4 \text{ ft/s}^2 \rightarrow$ ◀

$\mathbf{a}_B = 29.4 \text{ ft/s}^2 \rightarrow$ ◀

$\mathbf{a}_C = 30 \text{ ft/s}^2 \rightarrow$ ◀

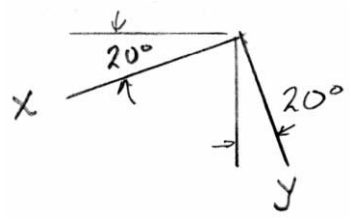
Since $a_A$, $a_B$, and $a_C$ are to the right, the friction forces $F_A$, $F_B$, and $F_C$ are to the left as assumed.

(*b*) Tension in the cable. $T = 5.56$ lb ◀

(*c*) Force **P**. $\mathbf{P} = 44.9$ lb $\rightarrow$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

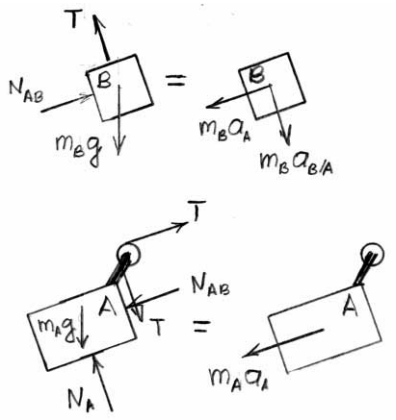
## Chapter 12, Solution 34.




Let the positive direction of $x$ and $y$ be those shown in the sketch, and let the origin lie at the cable anchor.

Constraint of cable: $x_A + y_{B/A} = \text{constant}$ or $a_A + a_{B/A} = 0$, where the positive directions of $a_A$ and $a_{B/A}$ are respectively the $x$ and the $y$ directions. Then $a_{B/A} = -a_A$ (1)

First note that $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = (a_A \angle 20°) + (a_{B/A} \angle 20°)$




Block $B$: $+\nearrow \Sigma F_x = m_B (a_B)_x$: $m_B g \sin 20° - N_{AB} = m_B a_A$

$$m_B a_A + N_{AB} = m_B g \sin 20°$$

$$15\, a_A + N_{AB} = 50.328 \quad (2)$$

$+\searrow \Sigma F_y = m_B (a_B)_y$: $m_B g \cos 20° - T = m_B a_{B/A}$

$$m_B a_{B/A} + T = m_B g \cos 20°$$

$$15\, a_{B/A} + T = 138.276 \quad (3)$$

Block $A$: $+\nearrow \Sigma F_x = m_A a_A$: $m_A g \sin 20° + N_{AB} + T = m_A a_A$

$$m_A a_A - N_{AB} + T = m_A g \sin 20°$$

$$25\, a_A - N_{AB} + T = 83.880 \quad (4)$$

Eliminate $a_{B/A}$ using Eq. (1), then add Eq. (4) to Eq. (2) and subtract Eq. (3).

$55\, a_A = -4.068$ or $a_A = -0.0740 \text{ m/s}^2$, $\mathbf{a}_A = 0.0740 \text{ m/s}^2 \angle 20°$ ◀

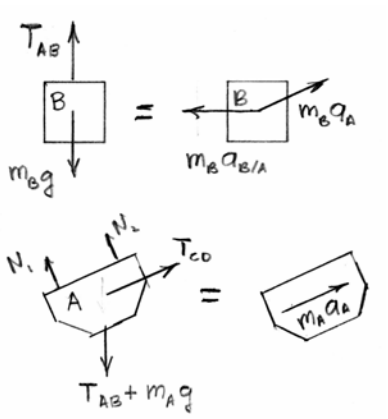
From Eq. (1), $a_{B/A} = 0.0740 \text{ m/s}^2$

From Eq. (3), $T = 137.2 \text{ N}$ $T = 137.2 \text{ N}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 35.

Motion of $B$ relative to $A$. Particle $B$ is constrained to move on a circular path with its center at point $A$. $(\mathbf{a}_{B/A})_t$ is the component of $\mathbf{a}_{B/A}$ lying along the circle, say to the left in the diagram and $(\mathbf{a}_{B/A})_n$ is directed toward point $A$. Initially, $\mathbf{a}_{B/A} = 0$, since the system starts from rest.




(*a*) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = (a_A \measuredangle 25°) + (a_{B/A} \leftarrow)$

Crate $B$: $\overset{+}{\leftarrow} \Sigma F_x = \Sigma m a_x$: $0 = m_B a_{B/A} - m_B a_A \cos 25°$

$$a_{B/A} = a_A \cos 25° = 0.4 \cos 25° = 0.363$$

$$\mathbf{a}_{B/A} = 0.363 \text{ m/s}^2 \leftarrow \blacktriangleleft$$

$$+\uparrow F_y = m_B (a_B)_y: \quad T_{AB} - m_B g = m_B a_A \sin 25°$$

$$T_{AB} = m_B (g + a_A \sin 25°)$$

$$= 250(9.81 + 0.4 \sin 25°) = 2495 \text{ N}$$

(*b*) Trolley $A$: $+\nearrow \Sigma F = m_A a_A$:

$$T_{CD} - (T_{AB} + m_A g) \sin 25° = m_A a_A$$

$$T_{CD} = (T_{AB} + m_A g) \sin 25° + m_A a_A$$

$$= \left[2495 + (20)(9.81)\right] \sin 25° + (20)(0.4)$$

$$T_{CD} = 1145 \text{ N} \blacktriangleleft$$

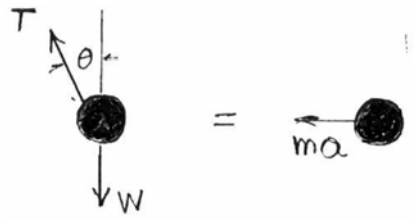

© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 36.

The ball moves at constant speed on a circle of radius

$$\rho = L\sin\theta$$

Acceleration (toward center of circle). $a = \dfrac{v^2}{\rho}$




$+\uparrow \quad \Sigma F_y = ma_y$: $\quad T\cos\theta - W = 0 \quad T = \dfrac{W}{\cos\theta}$

$+\leftarrow \quad \Sigma F_y = ma_x$: $\quad T\sin\theta = ma$

$$\frac{W}{\cos\theta}\sin\theta = \frac{mv^2}{\rho} = \frac{mv^2}{L\sin\theta}$$

(*a*) $\tan\theta\,\sin\theta = \dfrac{mv^2}{WL} = \dfrac{v^2}{gL} = \dfrac{(1.5)^2}{(9.81)(0.6)} = 0.38226$

$\theta = 34.21°$ $\qquad \theta = 34.2° \blacktriangleleft$

(*b*) $T = \dfrac{W}{\cos\theta} = \dfrac{mg}{\cos\theta} = \dfrac{(2)(9.81)}{\cos 34.21°}$ $\qquad T = 23.7\ \text{N} \blacktriangleleft$

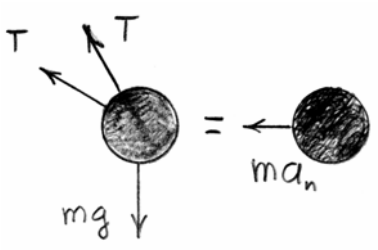
*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 37.

Let $\rho$ be the radius of the horizontal circle. The length of the wire is

$$L = \frac{\rho}{\sin\theta_1} + \frac{\rho}{\sin\theta_2}. \qquad \text{Solving for } \rho, \qquad \rho = \frac{L\sin\theta_1 \sin\theta_2}{\sin\theta_1 + \sin\theta_2}$$

$$\Sigma F_y = 0: \quad T\cos\theta_1 + T\cos\theta_2 - mg = 0$$




$$T = \frac{mg}{\cos\theta_1 + \cos\theta_2}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x: \quad T\sin\theta_1 + T\sin\theta_2 = ma_n = \frac{mv^2}{\rho}$$

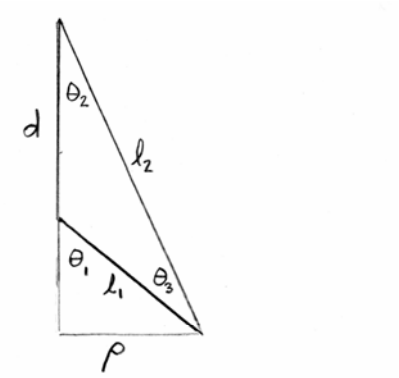
$$\frac{mg\left(\sin\theta_1 + \sin\theta_2\right)}{\cos\theta_1 + \cos\theta_2} = \frac{mv^2\left(\sin\theta_1 + \sin\theta_2\right)}{L\sin\theta_1\sin\theta_2}$$

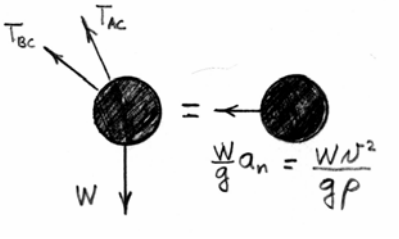
$$v^2 = Lg\frac{\sin\theta_1\sin\theta_2}{\cos\theta_1 + \cos\theta_2} = (2)(9.81)\frac{\sin 60^\circ \sin 30^\circ}{\cos 60^\circ + \cos 30^\circ} = 6.2193 \text{ m}^2/\text{s}^2$$

$$v = 2.49 \text{ m/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 38.




$$\theta_3 = \theta_1 - \theta_2 = 50^\circ - 25^\circ = 25^\circ$$

$$\frac{\ell_1}{\sin\theta_2} = \frac{d}{\sin\theta_3} \qquad \text{or} \qquad \ell_1 = \frac{d\sin\theta_2}{\sin\theta_3}$$

$$\rho = \ell_1 \sin\theta_1 = \frac{d\sin\theta_2\sin\theta_1}{\sin\theta_3}$$

$$= \frac{(4)(\sin 25^\circ)(\sin 50^\circ)}{\sin 25^\circ} = 3.0642 \text{ ft}$$

$$\Sigma F_y = 0: \quad T_{AC}\cos\theta_2 + T_{BC}\cos\theta_1 - W = 0 \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x: \quad T_{AC}\sin\theta_2 + T_{BC}\sin\theta_1 = \frac{Wv^2}{g\rho} \qquad (2)$$

Case 1: $T_{BC} = 0$. $\quad T_{AC}\cos\theta_2 - W = 0 \quad$ or $\quad T_{AC} = \dfrac{W}{\cos\theta_2}$

$$T_{AC}\sin\theta_2 = W\tan\theta_2 = \frac{Wv^2}{g\rho}$$

$$v^2 = g\rho\tan\theta_2 = (32.2)(3.0642)\tan 25^\circ = 46.01 \text{ ft}^2/\text{s}^2$$

$$v = 6.78 \text{ ft/s}$$

Case 2: $T_{AC} = 0$. $\quad T_{BC}\cos\theta_1 - W = 0 \quad$ or $\quad T_{BC} = \dfrac{W}{\cos\theta_1}$

$$T_{BC}\sin\theta_1 = W\tan\theta_1 = \frac{Wv^2}{g\rho}$$

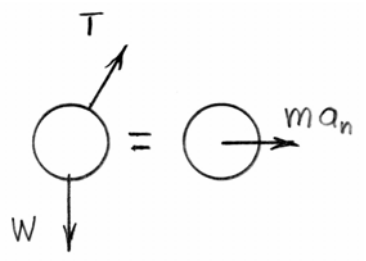
$$v^2 = g\rho\tan\theta_1 = (32.2)(3.0642)\tan 50^\circ = 117.59 \text{ ft}^2/\text{s}^2$$

$$v = 10.84 \text{ ft/s}$$

$$6.78 \text{ ft/s} \le v \le 10.84 \text{ ft/s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 39.




(*a*) $\Sigma F_y = 0: \quad T\sin\theta - W = 0$

$$T = \frac{W}{\sin\theta} = \frac{16}{\sin 60^\circ} \qquad \text{or } T = 18.48 \text{ lb} \blacktriangleleft$$

(*b*) $\overset{+}{\rightarrow} \Sigma F_x = ma_n: \quad T\cos\theta = m\frac{v^2}{\rho}$

$$v^2 = \frac{\rho T\cos\theta}{m} = \frac{\rho W\cos\theta}{m\sin\theta}$$

$$= \frac{\rho g}{\tan\theta} = \frac{(3)(32.2)}{\tan 60^\circ} = 55.77 \text{ ft}^2/\text{s}^2$$

$$v = 7.47 \text{ ft/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 40.

$$y = \frac{r^2}{2}, \quad \frac{dy}{dx} = r = 1 = \tan\theta \qquad \text{or} \qquad \theta = 45°$$

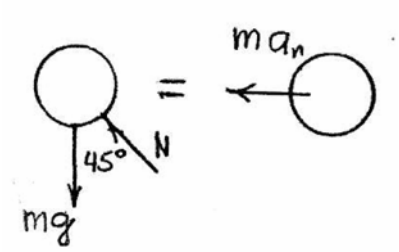
$$\Sigma F_y = 0: \quad N\cos\theta - mg = 0$$

$$N = \frac{mg}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x: \quad N\sin\theta = ma_n$$

$$mg\tan\theta = m\frac{v^2}{r}$$

$$v^2 = gr\tan\theta$$

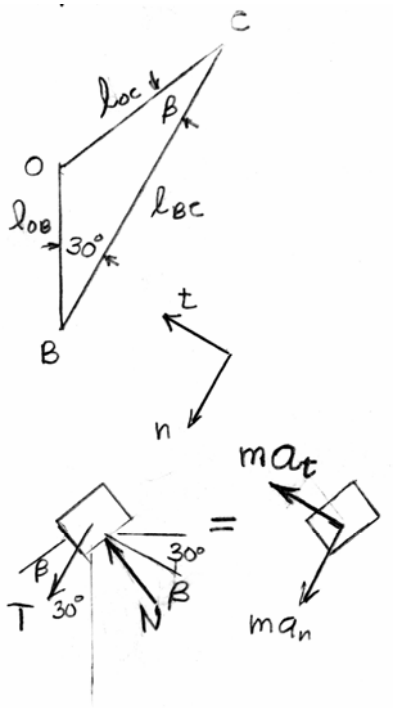



(*a*) $v^2 = (9.81)(1)(1.0000) = 9.81 \text{ m}^2/\text{s}^2$ $\qquad v = 3.13 \text{ m/s}$ ◀

(*b*) $N = \dfrac{mg}{\cos 45°} = \dfrac{(1)(9.81)}{\cos 45°}$ $\qquad N = 13.87 \text{ N}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 41.




Geometry

$$\ell_{OC}^2 = \ell_{OB}^2 + \ell_{BC}^2 - 2\ell_{OB}\ell_{OC}\cos 30°$$

$$= (0.3)^2 + (0.6)^2 - (2)(0.3)(0.6)\cos 30° = 0.13823 \text{ m}^2$$

$$\ell_{OC} = 0.37179 \text{ m}$$

$$\frac{\ell_{OC}}{\sin 30°} = \frac{\ell_{OB}}{\sin \beta}$$

$$\sin \beta = \frac{\ell_{OB} \sin 30°}{\ell_{OC}} = \frac{(0.3)\sin 30°}{0.37179} = 0.40345$$

$$\beta = 23.79°$$

Acceleration components: $a_t = 1.3 \text{ m/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{v_C^2}{\ell_{BC}} = \frac{(1)^2}{0.6} = 1.667 \text{ m/s}^2$$

Mass $m = 1 \text{ kg}$

$$+\nwarrow \Sigma F_t = ma_t: \quad N\cos\beta = (1)(1.3) = 1.3$$

$$N = \frac{1.3}{\cos 23.79°} = 1.421 \text{ N}$$

$$+\swarrow \Sigma F_n = ma_n: \quad T - N\sin\beta = (1)(1.667) = 1.667$$

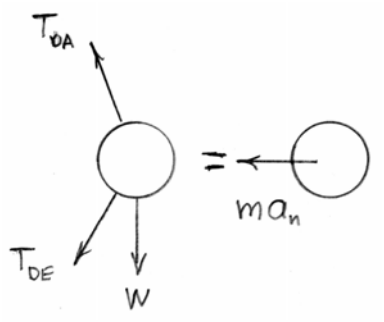
(*a*) $T = 1.667 + 1.421 \sin 23.79°$ $\qquad T = 2.24 \text{ N}$ ◀

(*b*) Force exerted by rod on collar is $1.421 \text{ N} \searrow (30° + \beta) = 53.8°$

Force exerted by collar on rod: $1.421 \text{ N} \nwarrow 53.8°$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 42.




$$W = mg = (0.5)(9.81) = 4.905 \text{ N}$$

$$\rho = 150 \text{ mm} = 0.150 \text{ m}$$

$$\Sigma F_y = 0: \quad T_{DA}\cos 20° - T_{DE}\cos 30° - W = 0$$

$$0.93969\, T_{DA} - 0.86603\, T_{DE} = 4.905$$

$$T_{DA} = 0.92160\, T_{DE} + 5.2198 \tag{1a}$$

$$T_{DE} = 1.08506\, T_{DA} - 5.6638 \tag{1b}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_n = \frac{mv^2}{\rho}: \quad T_{DA}\sin 20° + T_{DE}\sin 30° = \frac{0.5}{0.150}v^2$$

$$v^2 = 0.10261\, T_{DA} + 0.15\, T_{DE} \tag{2}$$

Try $T_{DA} = 75$ N. By Eq. (1*b*), $T_{DE} = 75.72$ N $> 75$ N (unacceptable)

Try $T_{DE} = 75$ N. By Eq. (1*a*), $T_{DA} = 74.34$ N $< 75$ N (acceptable)

By Eq. (2), $v^2 = (0.10261)(74.34) + (0.15)(75) = 18.877 \text{ m}^2/\text{s}^2$

$$v = 4.34 \text{ m}$$

Try $T_{DA} = 0$. By Eq. $(1b)$, $T_{DE} = -5.6638$ (unacceptable)

Try $T_{DE} = 0$. By Eq. $(1a)$, $T_{DA} = 5.2198$ (acceptable)

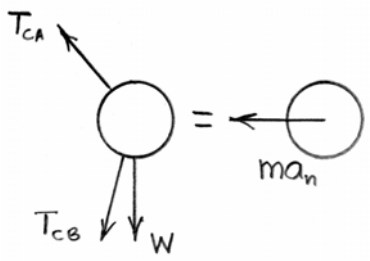
By Eq. (2), $v^2 = (0.10261)(5.2198) + (0.15)(0) = 0.5356 \text{ m}^2/\text{s}^2$

$$v = 0.732 \text{ m/s}$$

For $0 \le T_{BA}, T_{BC}, T_{DA}, T_{DE} \le 75$ N, $\quad 0.732 \text{ m/s} \le v \le 4.34 \text{ m/s}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 43.




$$W = mg = (5)(9.81) = 49.05 \text{ N}$$

$$\rho = 0.9 \text{ m}$$

$$\Sigma F_y = 0: \quad T_{CA}\cos 40° - T_{CB}\cos 15° - W = 0$$

$$0.76604\,T_{CA} - 0.96593\,T_{CB} = W = 49.05$$

$$T_{CB} = 0.79307\,T_{CA} - 50.780 \tag{1a}$$

$$T_{CA} = 1.26093\,T_{CB} + 64.030 \tag{1b}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x: \quad T_{CA}\sin 40° + T_{CB}\sin 15° = ma_n = \frac{mv^2}{\rho}$$

$$v^2 = \frac{\rho}{m}\left(T_{CA}\sin 40° + T_{CB}\sin 15°\right)$$

$$= \frac{0.9}{5}\left(T_{CA}\sin 40° + T_{CB}\sin 15°\right)$$

$$= 0.115702\,T_{CA} + 0.046587\,T_{CB} \tag{2}$$

Try $T_{CB} = 116$ N. By (1*b*), $T_{CA} = 210.3$ N (unacceptable)

Try $T_{CA} = 116$ N. By (1*a*), $T_{CB} = 41.216$ N (acceptable)

By (2), $v^2 = (0.115702)(116) + (0.046587)(41.216) = 15.34 \text{ m}^2/\text{s}^2$

$v = 3.92$ m/s

Try $T_{CA} = 0$. By $(1a)$, $T_{CB} = -50.78$ N (unacceptable)

Try $T_{CB} = 0$. By $(1b)$, $T_{CA} = 64.03$ N (acceptable)

By (2), $v^2 = (0.115702)(64.030) + 0 = 7.408 \text{ m}^2/\text{s}^2$

$v = 2.72$ m/s

For $0 \le T_{CA}, T_{CB} \le 116$ N, $\quad 2.72 \text{ m/s} \le v \le 3.92 \text{ m/s}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 44.

(*a*) Before wire *AB* is cut. $a = 0$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad -T_{AB}\cos 50° + T_{CD}\cos 70° = 0 \tag{1}$$

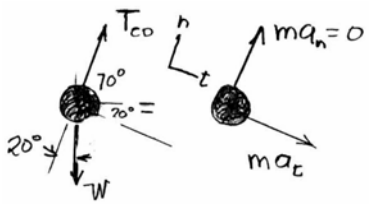
$$+\uparrow \Sigma F_y = 0: \quad T_{AB}\sin 50° + T_{CD}\sin 70° - W = 0 \tag{2}$$

Solving (1) and (2) simultaneously,

$$T_{AB} = 0.395\,W \qquad T_{CD} = 0.742\,W \blacktriangleleft$$

(*b*) Immediately after wire *AB* is cut. $T_{AB} = 0,\ v = 0$

$$a_n = \frac{v^2}{\rho} = 0$$




$$+\nearrow F_n = ma_n = 0:$$

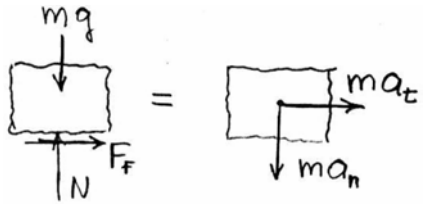
$$T_{CD} - W\cos 20° = 0$$

$$T_{CD} = W\cos 20°$$

$$T_{CD} = 0.940\,W \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 45.




$$+\uparrow \Sigma F_y = ma_y:$$

$$N - mg = -ma_n = -\frac{mv^2}{\rho}$$

$$N = mg - \frac{mv^2}{\rho}$$

$$\xrightarrow{+} \Sigma F_x = ma_x:$$

$$F_t = ma_t$$

At onset of slipping,

$$F_t = \mu_s N$$

$$ma_t = \mu_s\left(mg - \frac{mv_s^2}{\rho}\right)$$

$$v_s^2 = \rho\left(g - \frac{a_t}{\mu_s}\right) = (0.300)\left(9.81 - \frac{0.150}{0.75}\right) = 2.883 \text{ m}^2/\text{s}^2$$

$$v_s = 1.6979 \text{ m/s}$$

Time at slipping.

$$t_s = \frac{v_s}{a_t} = \frac{1.6979}{0.150}$$

$$t_s = 11.32 \text{ s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 46.**

Angle change over arc $AB$. $\theta = \dfrac{8^\circ}{180^\circ}\pi = 0.13963$ rad

Length of arc: $s_{AB} = \rho\theta = (4)(5280)(0.13963) = 2949$ ft

$s_{BC} = (0.5)(5280) = 2640$ ft, $s_{AC} = 2949 + 2640 = 5589$ ft

$$\int_{540}^{480} v\,dv = \int_0^{5589} a_t\,ds \qquad \text{or} \qquad \frac{480^2}{2} - \frac{540^2}{2} = a_t(5589)$$

$$a_t = -5.475 \text{ ft/s}^2$$

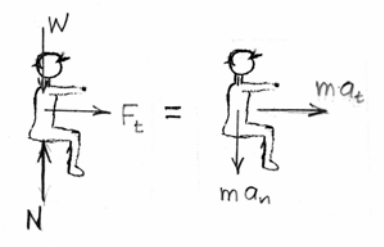
$$\int_{540}^{v_B} v\,dv = \int_0^{2949} a_t\,ds \qquad \text{or} \qquad \frac{v_B^2}{2} - \frac{540^2}{2} = (-5.475)(2949)$$

$$v_B^2 = 259300 \text{ ft}^2/\text{s}^2 \qquad v_B = 509.2 \text{ ft/s}$$

Mass of passenger: $m = \dfrac{200}{32.2} = 6.211$ lb·s$^2$/ft

Just before point $B$. $v = 509.2$ ft/s, $\rho = (4)(5280) = 21120$ ft

$$a_n = \frac{v^2}{\rho} = \frac{(509.2)^2}{21120} = 12.277 \text{ ft/s}^2$$




$\Sigma F_y = N_1 - W = -m(a_n)_1$: $N_1 = 200 - (6.211)(12.277) = 123.75$ lb

$\overset{+}{\rightarrow} \Sigma F_x = F_t = ma_t$: $(F_t)_1 = (0.6221)(-5.474) = -34.01$ lb

Just after point $B$. $v = 509.2$ ft/s, $\rho = \infty$, $a_n = 0$

$\Sigma F_y = 0$: $N_2 - W = 0$ $\qquad N_2 = W = 200$ lb

$\overset{+}{\rightarrow} \Sigma F_x = ma_t$: $F_t = ma_t = (6.211)(-5.475) = -34.01$ lb

$F_t$ does not change.

$N$ increases by 76.25 lb

magnitude of change of force = 76.3 lb ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 47.

$$s = 3\left(180t - t^2\right)\text{m}, \qquad v = \frac{ds}{dt} = 3\left(180 - 2t\right)\text{m/s},$$

$$a_t = \frac{dv}{dt} = -6\ \text{m/s}^2 \qquad \rho = (4)(5280) = 21120\ \text{ft}$$

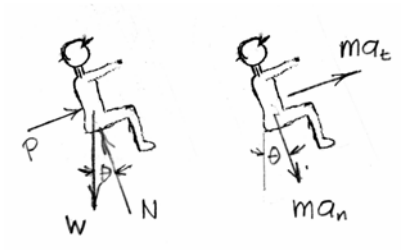
Length of arc $AB$. $s_{AB} = \rho\theta_{AB} = \left(21120\right)\dfrac{8°\pi}{180°} = 2949\ \text{ft}$

$$\int_{v_A}^{v_B} v\,dv = \int_0^{s_{AB}} a_t\,ds$$

$$\frac{v_B^2}{2} - \frac{v_A^2}{2} = 2a_t s_{AB} \quad \text{or} \quad v_B^2 = v_A^2 + 2a_t s_{AB} = 540^2 + \left(2\right)\left(-6\right)\left(2949\right)$$

$$= 256212, \qquad v_B = 506.2\ \text{ft/s}$$

For passenger, $W = 165$ lb, $m = \dfrac{W}{g} = \dfrac{165}{32.2} = 5.124\ \text{lb}\cdot\text{s}^2/\text{ft}$




$$+\searrow \Sigma F = ma_n: \quad W\cos\theta - N = \frac{mv^2}{\rho}$$

$$N = W\cos\theta - \frac{mv^2}{\rho} \tag{1}$$

$$+\nearrow \Sigma F = ma_t: \quad P - W\sin\theta = ma_t$$

$$P = W\sin\theta + ma_t \tag{2}$$

($a$) Just after point $A$, $t = 0$, $v = 180$ m/s, $\theta = 8°$

From Eq. (1), $N = 165\cos 8° - \dfrac{\left(5.124\right)\left(540\right)^2}{21120} = 92.65\ \text{lb}$

From Eq. (2), $P = 165\sin 8° + \left(5.124\right)\left(-6\right) = -7.78\ \text{lb}$

$F = \sqrt{N^2 + P^2} = 93.0\ \text{lb}, \quad \tan\beta = \dfrac{92.65}{7.78} = 11.909, \quad \beta = 85.2°$

$\beta - 8° = 77.2°$ $\qquad$ $\mathbf{F} = 93.0\ \text{lb}\ \searrow 77.2°$ ◀

($b$) At point $B$. $\theta = 0$, $v = 506.2$ ft/s

From Eq. (1), $N = 165 - \dfrac{\left(5.124\right)\left(506.2\right)^2}{21120} = 102.83\ \text{lb}$

From Eq. (2), $P = \left(5.124\right)\left(-6\right) = -30.74\ \text{lb}$

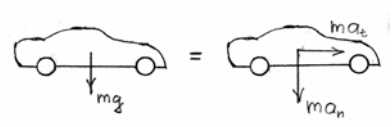
$F = \sqrt{\left(102.83\right)^2 + \left(30.74\right)^2} = 107.3\ \text{lb}, \quad \tan\beta = \dfrac{102.83}{30.74} = 3.345, \quad \beta = 73.4°$

$\mathbf{F} = 107.3\ \text{lb}\ \searrow 73.4°$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 48.

(*a*) $v = 160 \text{ km/h} = 44.44 \text{ m/s}$



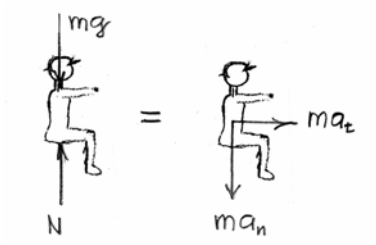

Wheels do not touch the road.

$$\Sigma F_y = -ma_n: \quad -mg = -mv^2/\rho$$

$$\rho = \frac{v^2}{g} = \frac{(44.44)^2}{9.81} = 201.4$$

$$\rho = 201 \text{ m} \blacktriangleleft$$

(*b*) $v = 80 \text{ km/h} = 22.22 \text{ m/s}$




$m = 70$ kg for passenger

$$\Sigma F_y = -ma_n:$$

$$N - mg = -\frac{mv^2}{\rho}$$

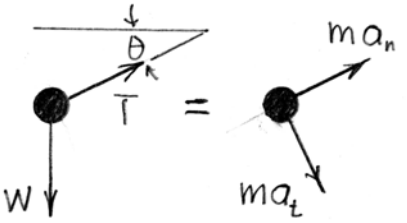
$$N = m\left(g - \frac{v^2}{\rho}\right)$$

$$= (70)\left(9.81 - \frac{22.22^2}{201.4}\right)$$

$$N = 515 \text{ N} \uparrow \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 49.




$$m = 0.2 \text{ kg}, \quad W = mg = (0.2)(9.81) = 1.962 \text{ N}$$

$$+\searrow \Sigma F_t = ma_t: \quad W\cos\theta = ma_t$$

$$a_t = \frac{W}{m}\cos\theta = g\cos\theta$$

$$\int_{v_0}^{v} v\,dv = \int_0^s a_t ds = \int_0^\theta a_t r d\theta$$

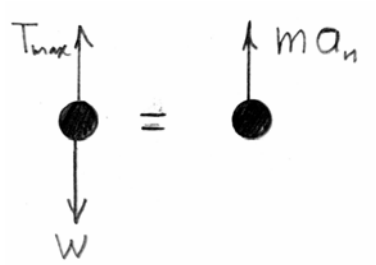
$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_0^\theta g\cos\theta r d\theta = gr\sin\theta$$

$$v^2 = v_0^2 + 2gr\sin\theta, \text{ where } r = 0.6 \text{ m for } 0 \le \theta \le 90°$$

$v = v_{\max}$ when the cord touches the peg or $\theta = 90°$.

$$v_{\max}^2 = v_0^2 + 2gr \tag{1}$$

When the cord touches the peg, the radius of curvature of the path becomes $\rho = 0.3$ m.




$$\Sigma F_y = ma_n: \quad T_{\max} - W = \frac{m{v_{\max}}^2}{\rho}$$

$${v_{\max}}^2 = \frac{\rho}{m}\left(T_{\max} - W\right) \tag{2}$$

Eliminating ${v_{\max}}^2$ from equations (1) and (2),

$$v_0^2 = \frac{\rho\left(T_{\max} - W\right)}{m} - 2gr = \frac{(0.3)(10 - 1.962)}{0.2} - (2)(9.81)(6.0)$$

$$= 0.285 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_0 = 0.534 \text{ m/s} \downarrow \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 50.**

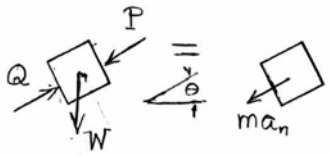
Mass of block $B$.

$$m_B = \frac{0.5\,\text{lb}}{32.2\,\text{ft/s}^2} = 0.015528\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Acceleration of block $B$.

$$a_n = \frac{v^2}{\rho} = \frac{(9\,\text{ft/s})^2}{3\,\text{ft}} = 27\,\text{ft/s}^2$$

$$a_t = 0 \qquad \text{since} \quad v = \text{constant}.$$




$$+\nearrow \sum F = ma:$$

$$Q - P - W\sin\theta = -ma_n$$

$$Q = P + W\sin\theta - ma_n$$

But, $Q \geq 0$.

$$P + W\sin\theta - ma_n > 0$$

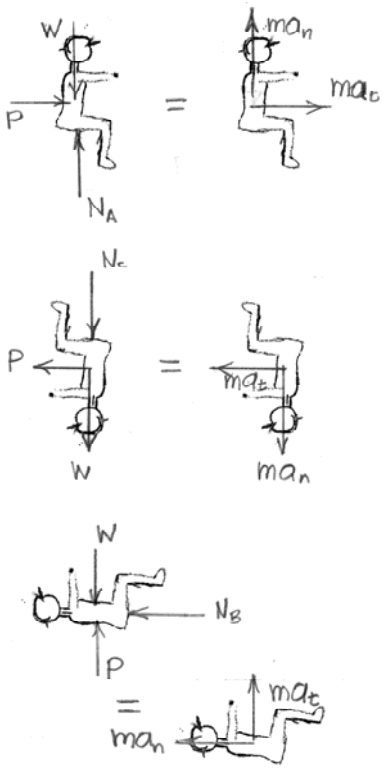
$$\sin\theta \geq \frac{ma_n}{W} - \frac{P}{W} = \frac{(0.015528)(27)}{0.5} - \frac{0.3}{0.5}$$

$$\sin\theta \geq 0.2385$$

$$13.8° \leq \theta \leq 166.2° \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 51.




At position $A$, the vertical component of apparent weight is shown as $N_A$.

$$\Sigma F = ma_n: \quad N_A - W = \frac{W}{g}a_n$$

$$a_n = \frac{N_A - W}{W}g = \frac{380 - 120}{120}(32.2) = 69.77 \text{ ft/s}^2$$

$$v_A^2 = \rho a_n = (3600)(69.77) = 251.2 \times 10^3 \text{ ft}^2/\text{s}^2$$

At position $C$, the vertical component of apparent weight is shown as $N_C$.

$$+\downarrow \Sigma F = ma_n: \quad N_C + W = \frac{W}{g}a_n$$

$$a_n = \frac{N_C + W}{W}g = \frac{80 + 120}{120}(32.2) = 53.67 \text{ ft/s}^2$$

$$v_C^2 = \rho a_n = (3600)(53.67) = 193.2 \times 10^3 \text{ ft}^2/\text{s}^2$$

Length of arc $ABC$:

$$s_{AC} = \pi\rho = \pi(3600) = 11310 \text{ ft}$$

Calculate $a_t$, using $v_C^2 - v_A^2 = 2a_t s_{AC}$

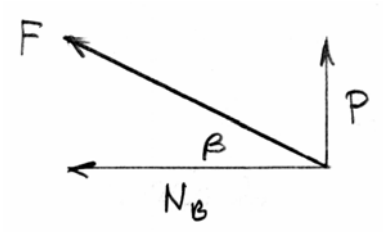
$$a_t = \frac{v_C^2 - v_A^2}{2s_{AC}} = \frac{193.2 \times 10^3 - 251.2 \times 10^3}{(2)(11310)}$$

$$= -2.562 \text{ ft/s}^2$$

At position $B$, $s_{AB} = \frac{\pi}{2}\rho = \frac{\pi}{2}(3600) = 5655$ ft

$$v_B^2 = v_A^2 + 2a_t s_{AB} = 251.2 \times 10^3 + (2)(-2.562)(5655) = 222.2 \times 10^3 \text{ ft}^2/\text{s}^2$$


© 2007 The McGraw-Hill Companies.




Effective forces at $B$: $ma_n = \dfrac{W}{g}\dfrac{v_B^2}{\rho} = \dfrac{120}{32.2}\dfrac{222.2 \times 10^3}{3600} = 230$ lb

$$ma_t = \frac{W}{g}a_t = \frac{120}{32.2}(-2.562) = -9.5 \text{ lb}$$

$\Sigma F = ma_t$:

$$P - W = ma_t \text{ or } P = W + ma_t = 120 - 9.5 = 110.5 \text{ lb}$$

$+\leftarrow \Sigma F = ma_n$: $N_B = ma_n = 230$ lb

Force exerted by seat:

$$F = \sqrt{N_B{}^2 + P^2} = \sqrt{230^2 + 110.5^2} = 255 \text{ lb}$$

$$\tan\beta = \frac{110.5}{230} \qquad \beta = 25.7°$$

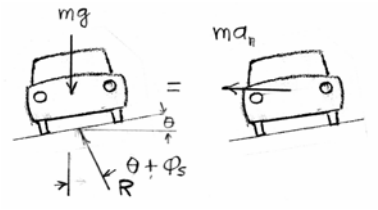
**F** = 255 lb ∠ 25.7° ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 52.

The road reaction consists of normal component **N** and friction component **F**. The resultant **R** makes angle $\phi_s$ with the normal.

Case 1: $v = v_{\max}$




$$\Sigma F_y = 0: \quad R\cos(\theta + \phi_s) - mg = 0$$

$$R = \frac{mg}{\cos(\theta + \phi_s)}$$

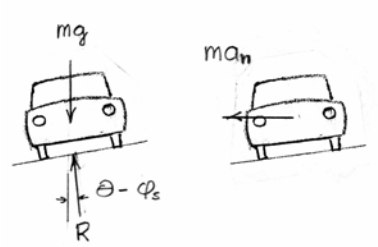
$$\overset{+}{\leftarrow} \Sigma F_x = ma_n: \quad R\sin(\theta + \phi_s) = ma_n$$

$$ma_n = mg\tan(\theta + \phi_s)$$

$$\frac{v_{\max}^2}{r} = g\tan(\theta + \phi_s)$$

$$v_{\max} = \sqrt{gr\tan(\theta + \phi_s)}$$

Case 2: $v = v_{\min}$




$$\Sigma F_y = 0: \quad R\cos(\theta - \phi_s) - mg = 0$$

$$R = \frac{mg}{\cos(\theta - \phi_s)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_n: \quad R\sin(\theta - \phi_s) = ma_n$$

$$ma_n = mg\tan(\theta - \phi_s)$$

$$\frac{v_{\min}^2}{r} = g\tan(\theta - \phi_s)$$

$$v_{\min} = \sqrt{gr\tan(\theta - \phi_s)}$$

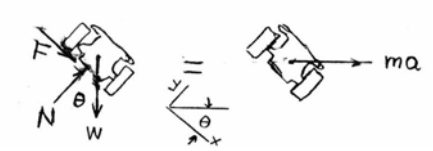
$$\sqrt{gr\tan(\theta - \phi_s)} \le v \le \sqrt{gr\tan(\theta + \phi_s)} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 53.**

Weight. $W = mg$

Acceleration. $a = \dfrac{v^2}{\rho}$




$\sum F_x = ma_x$:

$$F + W\sin\theta = ma\cos\theta$$

$$F = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta \qquad (1)$$

$\sum F_y = ma_y$:

$$N - W\cos\theta = ma\sin\theta$$

$$N = \frac{mv^2}{\rho}\sin\theta + mg\cos\theta \qquad (2)$$

(*a*) Banking angle. Rated speed $v = 180$ km/h $= 50$ m/s. $F = 0$ at rated speed.

$$0 = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta$$

$$\tan\theta = \frac{v^2}{\rho g} = \frac{(50)^2}{(200)(9.81)} = 1.2742$$

$$\theta = 51.875° \qquad \theta = 51.9° \blacktriangleleft$$

(*b*) Slipping outward. $v = 320$ km/h $= 88.889$ m/s

$F = \mu N$

$$\mu = \frac{F}{N} = \frac{v^2\cos\theta - \rho g\sin\theta}{v^2\sin\theta + \rho g\cos\theta}$$

$$\mu = \frac{(88.889)^2\cos 51.875° - (200)(9.81)\sin 51.875°}{(88.889)^2\sin 51.875° + (200)(9.81)\cos 51.875°}$$

$$= 0.44899 \qquad \mu = 0.449 \blacktriangleleft$$

*continued*

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

(*c*) Minimum speed. $F = -\mu N$

$$-\mu = \frac{v^2 \cos\theta - \rho g \sin\theta}{v^2 \sin\theta + \rho g \cos\theta}$$

$$v^2 = \frac{\rho g (\sin\theta - \mu \cos\theta)}{\cos\theta + \mu \sin\theta}$$

$$= \frac{(200)(9.81)(\sin 51.875° - 0.44899 \cos 51.875°)}{\cos 51.875° + 0.44899 \sin 51.875°}$$

$$= 1029.87 \text{ m}^2/\text{s}^2$$

$$v = 32.09 \text{ m/s}$$

$v = 115.5$ km/h ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 54.

Rated speed: $v_R = 75$ mi/h $= 110$ ft/s, $125$ mi/h $= 183.33$ ft/s

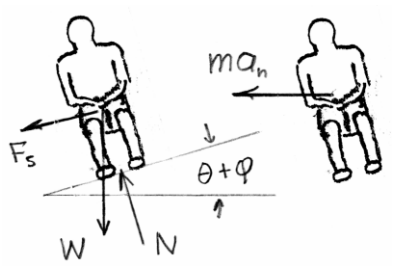
From Sample Problem 12.6,

$$v_R^2 = g\rho \tan\theta \qquad \text{or} \qquad \rho = \frac{v_R^2}{g \tan\theta} = \frac{(110)^2}{32.2 \tan 8^\circ} = 2674 \text{ ft}$$

Let the $x$-axis be parallel to the floor of the car.

$$+\!\!\nearrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n \cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$




($a$) $\phi = 0$.

$$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

$$= W\left[\frac{(183.33)^2}{(32.2)(2674)}\cos 8^\circ - \sin 8^\circ\right]$$

$$= 0.247\,W \qquad F_s = 0.247\,W \blacktriangleleft$$

($b$) For $F_s = 0$,

$$\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi) = 0$$

$$\tan(\theta+\phi) = \frac{v^2}{g\rho} = \frac{(183.33)^2}{(32.2)(2674)} = 0.39035$$

$$\theta + \phi = 21.3^\circ$$

$$\phi = 21.3^\circ - 8^\circ \qquad \phi = 13.3^\circ \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 55.

Rated speed: $v_R = 75$ mi/h $= 110$ ft/s, $125$ mi/h $= 183.33$ ft/s

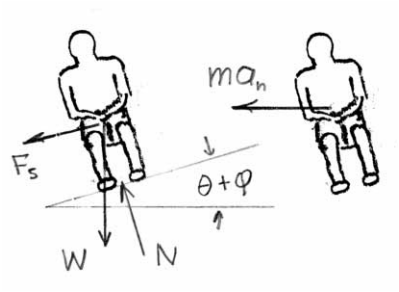
From Sample Problem 12.6,

$$v_R^2 = g\rho \tan\theta \qquad \text{or} \qquad \rho = \frac{v_R^2}{g \tan\theta} = \frac{(110)^2}{32.2 \tan 8^\circ} = 2674 \text{ ft}$$

Let the $x$-axis be parallel to the floor of the car.

$$+\nearrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n \cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$




Solving for $F_s$, $\quad F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$

Now $\frac{v^2}{g\rho} = \frac{(183.33)^2}{(32.2)(2674)} = 0.39035$ and $F_s = 0.12\,W$ so that

$$0.12\,W = W\left[0.39035\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

Let $u = \sin(\theta+\phi)$. Then, $\quad \cos(\theta+\phi) = \sqrt{1-u^2}$.

$$0.12 = 0.39035\sqrt{1-u^2} - u \qquad \text{or} \qquad 0.39035\sqrt{1-u^2} = 0.12 + u$$

Squaring both sides, $\quad 0.15237\left(1-u^2\right) = 0.0144 + 0.24u + u^2$

or $\quad 1.15237u^2 + 0.24u + 0.13797 = 0$

The positive root of the quadratic equation is $u = 0.2572$.

Then, $\theta + \phi = \sin^{-1} u = 14.90^\circ$

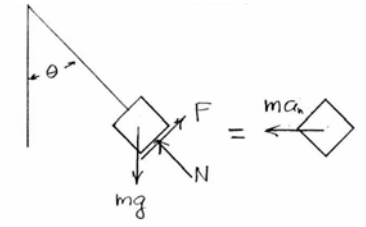
$\phi = 14.90^\circ - \theta = 14.90^\circ - 8^\circ$ $\qquad \phi = 6.90^\circ$ ◀


© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 56.**

If the collar is not sliding, it moves at constant speed on a circle of radius $\rho = r\sin\theta$. $v = \rho\omega$

Normal acceleration. $a_n = \dfrac{v^2}{\rho} = \dfrac{\rho^2\omega^2}{\rho} = (r\sin\theta)\omega^2$




$$+\uparrow \Sigma F_y = ma_y:$$

$$N\cos\theta - mg = 0$$

$$N = \frac{mg}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x:$$

$$N\sin\theta = ma$$

$$\frac{mg\sin\theta}{\cos\theta} = (m\ r\sin\theta)\omega^2$$

Either $\sin\theta = 0$

or $\cos\theta = \dfrac{g}{r\omega^2}$

$\theta = 0°$ or $180°$

or $\cos\theta = \dfrac{9.81}{(0.5)(7.5)^2} = 0.3488$

$\theta = 0°, 180°$, and $69.6°$ ◀


© 2007 The McGraw-Hill Companies.

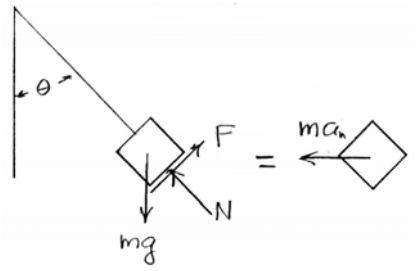
**Chapter 12, Solution 57.**

If the collar is not sliding, it moves at constant speed on a circle of radius $\rho = r\sin\theta$. $v = \rho\omega$

From Prob. 12.56 $r = 500\,\text{mm} = 0.500\,\text{m}$, $\omega = 7.5\,\text{rad/s}$,

$m = 250\,\text{g} = 0.250$ kg.

Normal acceleration: $a_n = \dfrac{v^2}{\rho} = \dfrac{\rho^2\omega^2}{\rho} = (r\sin\theta)\omega^2$




$+\nearrow \Sigma F = \nearrow ma$:

$F - mg\sin\theta = -m(r\sin\theta)\,\omega^2\cos\theta$

$F = m(g - r\omega^2\cos\theta)\sin\theta$

$+\nwarrow \Sigma F = \nwarrow ma$:

$N - mg\cos\theta = m(r\sin\theta)\omega^2\sin\theta$

$N = m(g\cos\theta) + r\,\omega^2\sin^2\theta)$

$(a)$ $\theta = 75°$. $F = (0.25)\left[9.81 - (0.500\cos 75°)(7.5)^2\right]\sin 75°$

$= 0.61112$ N

$N = (0.25)\left[9.81\cos 75° + (0.500\sin^2 75°)(7.5)^2\right]$

$= 7.1950$ N

$\mu_s N = (0.25)(7.1950) = 1.7987$ N

Since $|F| < \mu_s N$, the collar does not slide.

$F = 0.611$ N $\measuredangle 75°$ ◀

*continued*


© 2007 The McGraw-Hill Companies.

(*b*) $\theta = 40°$.

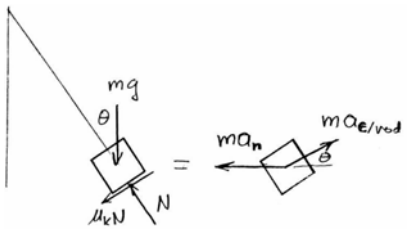
$$F = (0.25)\left[9.81 - (0.500\cos 40°)(7.5)^2\right]\sin 40°$$
$$= -1.8858 \text{ N}$$

$$N = (0.25)\left[9.81\cos 40° + (0.500\sin^2 40°)(7.5)^2\right]$$
$$= 4.7839 \text{ N}$$

$$\mu_s N = (0.25)(4.7839) = 1.1960 \text{ N}$$

Since $|F| > \mu_s N$, the collar slides.

Since the collar is sliding, $|F| = \mu_k N$.




$$+\nwarrow \Sigma F_n = +\nwarrow ma:$$

$$N - mg\cos\theta = ma_n \sin\theta$$

$$N = mg\cos\theta + m(r\sin\theta)\omega^2 \sin\theta$$

$$= m\left[g\cos\theta + (r\sin^2\theta)\omega^2\right]$$

$$= (0.25)\left[9.81\cos 40° + (0.500\sin^2 40°)(7.5)^2\right]$$

$$= 4.7839 \text{ N}$$

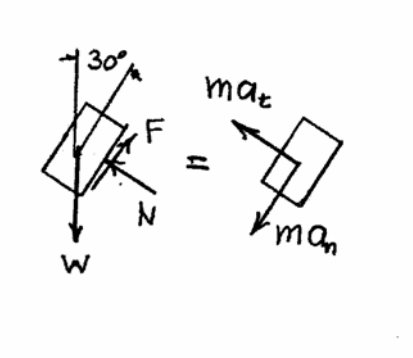
$$|F| = \mu_k N = (0.20)(4.7839) = 0.957 \text{ N}$$

$F = 0.957$ N $\swarrow$ 40° ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 58.

Draw the free body diagrams of the block $B$ when the arm is at $\theta = 150°$.




$$\dot{v} = a_t = 0, \quad g = 32.2 \text{ ft/s}^2$$

$$+\nwarrow \Sigma F_t = ma_t: \quad -W\sin 30° + N = 0$$

$$N = W\sin 30°$$

$$+\swarrow \Sigma F_n = ma_n: \quad W\cos 30° - F = m\frac{v^2}{\rho} = \frac{Wv^2}{\rho g}$$

$$F = W\cos 30° - \frac{Wv^2}{\rho g}$$

Form the ratio $\frac{F}{N}$, and set it equal to $\mu_s$ for impending slip.

$$\mu_s = \frac{F}{N} = \frac{\cos 30° - v^2/\rho g}{\sin 30°} = \frac{\cos 30° - (4.2)^2/(1)(32.2)}{\sin 30°}$$

$$\mu_s = 0.636 \blacktriangleleft$$

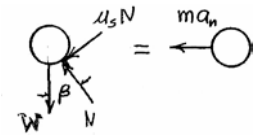

© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 59.

Let $\beta$ be the slope angle of the dish. $\tan\beta = \dfrac{dy}{dr} = \dfrac{1}{6}r$

At $r = 6$ft, $\tan\beta = 1$ or $\beta = 45°$

Draw free body sketches of the sphere.




$$\Sigma F_y = 0: \quad N\cos\beta - \mu_s N\sin\beta - W = 0$$

$$N = \frac{W}{\cos\beta - \mu_s\sin\beta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n: \quad N\sin\beta + \mu_s N\cos\beta = \frac{mv^2}{\rho} = \frac{Wv^2}{\rho g}$$

$$\frac{W\left(\sin\beta + \mu_s N\cos\beta\right)}{\cos\beta - \mu_s\sin\beta} = \frac{Wv^2}{\rho g}$$

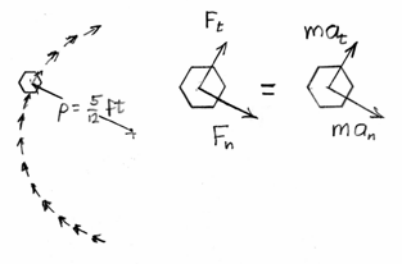
$$v^2 = \rho g\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = (6)(32.2)\frac{\sin 45° + 0.5\cos 45°}{\cos 45° - 0.5\sin 45°} = 579.6 \text{ ft}^2/\text{s}^2$$

$v = 24.1$ ft/s ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 60.

Uniformly accelerated motion on a circular path. $\rho = \dfrac{D}{2} = 5 \text{ in.} = \dfrac{5}{12} \text{ ft}$




$$W = 60 \times 10^{-6} \text{ oz} = \frac{60 \times 10^{-6}}{16} \text{lb}$$

$$g = 32.2 \text{ ft/s}^2$$

$$m = \frac{W}{g} = 116.46 \times 10^{-9} \text{lb s}^2/\text{ft}$$

(*a*) For uniformly accelerated motion,

$$v = v_0 + a_t t = 0 + (12)(4)$$

$$v = 48 \text{ ft/s} \blacktriangleleft$$

(*b*) $+\nearrow \Sigma F_t = ma_t: \quad F_t = \left(116.46 \times 10^{-9}\right)(12) = 1.3975 \times 10^{-6} \text{ lb.}$

$$+\searrow \Sigma F_n = ma_n: \quad F_n = ma_n = \frac{mv^2}{\rho} = \frac{\left(116.46 \times 10^{-9}\right)(48)^2}{5/12}$$

$$= 644.0 \times 10^{-6} \text{ lb}$$

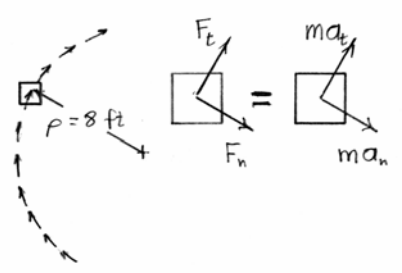
Magnitude of force:

$$F = \sqrt{F_t^2 + F_n^2} = 10^{-6}\sqrt{(644.0)^2 + (1.3975)^2}$$

$$F = 644 \times 10^{-6} \text{ lb} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 61.




Uniformly accelerated motion on a circular path. $\rho = 8$ ft

$$v = v_0 + a_t t$$

$$= 0 + (0.75)(12) = 9 \text{ ft/s}$$

$$F_t = ma_t = \frac{W}{g}a_t: \quad F_t = W\frac{a_t}{g} = \frac{0.75}{32.2}W = 0.0233\,W$$

$$F_n = ma_n = \frac{Wv^2}{g\rho}: \quad F_n = \frac{W(9)^2}{(32.2)(8)} = 0.3144\,W$$

$$F = \sqrt{F_t^2 + F_n^2} = 0.315\,W$$

This is the friction force available to cause the trunk to slide.

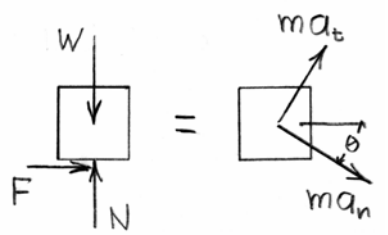
The normal force $N$ is calculated from equilibrium of forces in the vertical direction.

$$\Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

Since sliding is impending, $\mu_s = \frac{F}{W} = 0.315$ $\qquad \mu_s = 0.315$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 62.**




For constant speed, $a_t = 0$

$$a_n = \frac{v_B^2}{\rho} \text{ with } v_B = 0.7 \text{ m/s}, \rho = 0.2 \text{ m}$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad F = ma_n \cos\theta$$

$$\Sigma F_y = ma_y: \quad N - W = -ma_n \sin\theta \qquad N = mg - ma_n \sin\theta$$

Ratio $$\frac{F}{N} = \frac{ma_n \cos\theta}{mg - ma_n \sin\theta} = \frac{\cos\theta}{\frac{g}{a_n} - \sin\theta} = \frac{\cos\theta}{\frac{g\rho}{v_B^2} - \sin\theta}$$

With

$$\frac{g\rho}{v_B^2} = \frac{(9.81)(0.2)}{(0.7)^2} = 4.0041, \text{ the ratio becomes } \frac{F}{N} = \frac{\cos\theta}{4.0041 - \sin\theta}$$

For no impending slide, $$\mu_s \geq \frac{|F|}{N} = \frac{|\cos\theta|}{4.0041 - \sin\theta}$$

To find the value of $\theta$ for which the ratio is maximum set the derivative with respect to $\theta$ equal to zero.

$$\frac{d}{d\theta}\left(\frac{\pm\cos\theta}{4.0041 - \sin\theta}\right) = \pm\frac{1 - 4.0041\sin\theta}{(4.0041 - \sin\theta)^2} = 0$$

$$\sin\theta = \frac{1}{4.0041} = 0.24974$$

$$\theta = 14.446°, \quad \frac{|F|}{N} = \frac{|\cos 14.446°|}{4.0041 - 0.24974} = 0.258$$

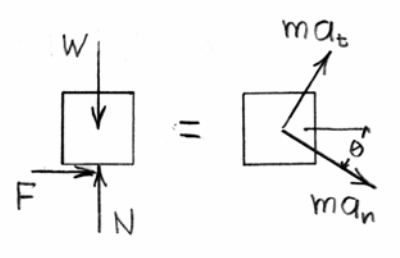
$$\theta = 180° - 14.446° = 165.554°, \quad \frac{|F|}{N} = 0.258$$

(*a*) Minimum value of $\mu_s$ for no slip. $(\mu_s)_{\min} = 0.258$ ◀

(*b*) Corresponding values of $\theta$. $\theta = 14.5°$ and $165.5°$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 63.




For constant speed, $a_t = 0$

$$a_n = \frac{v_B^2}{\rho} \text{ with } v_B = 0.7 \text{ m/s}, \ \rho = 0.2 \text{ m}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x: \quad F = ma_n \cos\theta$$

$$\Sigma F_y = ma_y: \quad N - W = -ma_n \sin\theta \qquad N = mg - ma_n \sin\theta$$

$$\text{Ratio } \frac{F}{N} = \frac{ma_n \cos\theta}{mg - ma_n \sin\theta} = \frac{\cos\theta}{\frac{g}{a_n} - \sin\theta} = \frac{\cos\theta}{\frac{g\rho}{v_B^2} - \sin\theta}$$

$$\text{Let } u = \frac{g\rho}{v_B^2} \text{ so that } \frac{F}{N} = \frac{\cos\theta}{u - \sin\theta}$$

Determine the value of $\theta$ at which $F/N$ is maximum.

$$\frac{d}{d\theta}\left(\frac{\cos\theta}{u - \sin\theta}\right) = \frac{\cos^2\theta - (u - \sin\theta)(\sin\theta)}{(u - \sin\theta)^2} = \frac{1 - u\sin\theta}{(u - \sin\theta)^2} = 0$$

The corresponding ratio $\frac{F}{N}$.

$$\frac{F}{N} = \frac{\pm\sqrt{1 - u^{-2}}}{u - u^{-1}} = \frac{\pm u^{-1}}{\sqrt{1 - u^{-2}}} = \pm\frac{\sin\theta}{\cos\theta} = \pm\tan\theta$$

(*a*) For impending sliding to the left: $\frac{F}{N} = \tan\theta = \mu_s = 0.35$

$$\theta = \arctan(0.35) = 19.29°, \ u^{-1} = \frac{v_B^2}{g\rho} = \sin\theta,$$

$$v_B^2 = (9.81)(0.2)\sin 19.29° = 0.648 \text{ m}^2/\text{s}^2$$

$$v_B = 0.805 \text{ m/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

For impending motion to the right: $\frac{F}{N} = -\tan\theta = \mu_s = 0.35$

$$\theta = \arctan(-0.35) = 160.71°$$

$$u^{-1} = \frac{v_2^2}{g\rho} = \sin\theta, \quad v_B^2 = (9.81)(0.2)\sin 160.71° = 0.648 \text{ m}^2/\text{s}^2$$

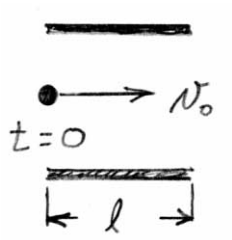
$$= 0.805 \text{ m/s} \blacktriangleleft$$

(*b*) For impending sliding to the left, $\theta = 19.3°$ ◀

For impending sliding to the right, $\theta = 160.7°$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 64.




Consider the motion of one electron. For the horizontal motion, let $x = 0$ at the left edge of the plate and $x = \ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t, \quad \text{so that} \quad t_1 = \frac{\ell}{v_0} \qquad \text{and} \qquad t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between plates $(0 \le t \le t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 \le t \le t_2)$, it is zero.

For $0 \le t \le t_1$, $\qquad \Sigma F_y = ma_y: \quad a_y = \dfrac{F_y}{m} = \dfrac{eV}{md}$

$$v_y = \left(v_y\right)_0 + a_y t = 0 + \frac{eVt}{md}$$

$$y = y_0 + \left(v_y\right)_0 t + \frac{1}{2} a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

At $t = t_1$, $\quad \left(v_y\right)_1 = \dfrac{eVt_1}{md} \quad$ and $\quad y_1 = \dfrac{eVt_1^2}{2md}$

For $t_1 \le t \le t_2$, $\quad a_y = 0$

$$y = y_1 + \left(v_y\right)_1 (t - t_1)$$

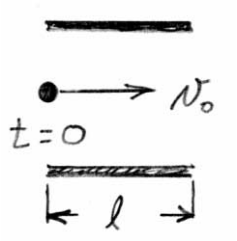
At $t = t_2$ $\qquad y_2 = \delta = y_1 + \left(v_y\right)_1 (t_2 - t_1)$

$$\delta = \frac{eVt_1^2}{2md} + \frac{eVt_1}{md}(t_2 - t_1) = \frac{eVt_1}{md}\left(t_2 - \frac{1}{2}t_1\right)$$

$$= \frac{eV\ell}{mdv_0}\left(\frac{\ell}{2v_0} + \frac{L}{v_0} - \frac{1}{2}\frac{\ell}{v_0}\right) \qquad \text{or} \quad \delta = \frac{eV\ell L}{mdv_0^2} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 65.




Consider the motion of one electron. For the horizontal motion, let $x = 0$ at the left edge of the plate and $x = \ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t, \quad \text{so that} \quad t_1 = \frac{\ell}{v_0} \qquad \text{and} \qquad t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between the plates $(0 \le t \le t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 \le t \le t_2)$, it is zero.

For $0 \le t \le t_1$,

$$\Sigma F_y = ma_y: \quad a_y = \frac{F_y}{m} = \frac{eV}{md}$$

$$v_y = \left(v_y\right)_0 + a_y t = 0 + \frac{eVt}{md}$$

$$y = y_0 + \left(v_y\right)_0 t + \frac{1}{2} a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

At $t = t_1, \ \frac{\ell}{v_0}, \ y = \frac{eV\ell^2}{2mdv_0^2}$

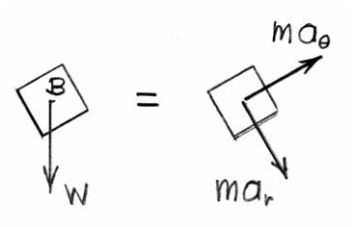
But $y < \frac{d}{2} - 0.075d = 0.425d$

So that $\frac{eV\ell^2}{2mdv_0^2} < 0.425d$

$$\frac{d^2}{\ell^2} > \frac{1}{0.425}\frac{eV}{2mv_0^2} = 1.176\frac{eV}{mv_0^2} \qquad \frac{d}{\ell} > 1.085\sqrt{\frac{eV}{mv_0^2}} \ \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 66.




$$\theta = 30°, \quad \dot{\theta} = 2 \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$r = 0.6 \text{ m}, \quad W = mg$$

Block $B$: Only force is weight

$$F_r = W\cos 30°, \quad F_\theta = -W\sin 30°$$

(*a*) $F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$:

$$2\dot{r}\dot{\theta} = \frac{F_\theta}{m} - r\ddot{\theta} = -\frac{mg\sin 30°}{m} - r\ddot{\theta} = -g\sin 30° - r\ddot{\theta}$$

$$\dot{r} = -\frac{g\sin 30° + r\ddot{\theta}}{2\dot{\theta}} = -\frac{(9.81)\sin 30° + (0.6)(0)}{(2)(2)} = -1.226 \text{ m/s}$$

$$\mathbf{v}_{B/\text{rod}} = 1.226 \text{ m/s} \measuredangle 60° \blacktriangleleft$$

(*b*) $F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$:

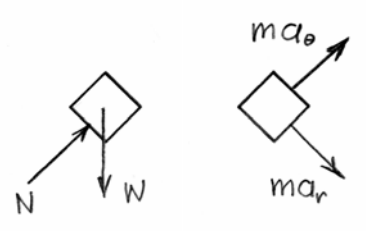
$$\ddot{r} = r\dot{\theta} + \frac{F_r}{m} = r\dot{\theta}^2 + \frac{mg\cos 30°}{m} = r\dot{\theta}^2 + g\cos 30°$$

$$= (0.6)(2)^2 + (9.81)\cos 30° = 10.90 \text{ m/s}^2$$

$$\mathbf{a}_{B/\text{rod}} = 10.90 \text{ m/s}^2 \measuredangle 60° \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 67.




$$\theta = 45°, \quad r = 0.8 \text{ m}, \quad \ddot{\theta} = 10 \text{ rad/s}^2$$

$$v_r = \dot{r} = 0, \quad v_\theta = r\dot{\theta} = 0, \quad W = mg$$

(*a*) $$\Sigma F_\theta = ma_\theta: \quad N - W\cos 45° = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$N = mg\cos 45° + m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$= (0.5)(9.81)\cos 45° + 0.5\left[(0.8)(10) + 0\right]$$

$$= 7.468 \qquad N = 7.47 \text{ N} \angle 45° \blacktriangleleft$$

(*b*) $$\Sigma F_r = ma_r: \quad mg\sin 45° = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\ddot{r} = \frac{mg}{m}\sin 45° + r\dot{\theta}^2 = g\sin 45° + r\dot{\theta}^2$$

$$= (9.81)\sin 45° + 0 = 6.937 \text{ m/s}^2$$

$$\mathbf{a}_{B/\text{rod}} = 6.94 \text{ m/s}^2 \angle 45° \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 68.**

Use radial and transverse components of acceleration.

$$r = 3t^2 - t^3 \text{ ft} \qquad \theta = 2t^2 \text{ rad}$$

$$\dot{r} = 6t - 3t^2 \text{ ft/s} \qquad \dot{\theta} = 4t \text{ rad/s}$$

$$\ddot{r} = 6 - 6t \text{ ft/s}^2 \qquad \ddot{\theta} = 4 \text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 6t - (3t^2 - t^3)(16t^2)$$

$$= 16t^5 - 48t^4 - 6t + 6 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (3t^2 - t^3)(4) + (2)(6t - 3t^2)(4t)$$

$$= 60t^2 - 28t^3 \text{ ft/s}^2$$

Mass: $m = \dfrac{W}{g} = \dfrac{4}{32.2} = 0.12422 \text{ lb}\cdot\text{s}^2/\text{ft}$

(*a*) $t = 0$. $a_r = 6 \text{ ft/sec}^2$

$a_\theta = 0$

Apply Newton's second law.

$F_r = ma_r = (0.12422)(6)$ $\qquad F_r = 0.745 \text{ lb}$ ◀

$F_\theta = ma_\theta = (0.12422)(0)$ $\qquad F_\theta = 0$ ◀

(*b*) $t = 1 \text{ s}$. $a_r = -32 \text{ ft/s}^2$

$a_\theta = 32 \text{ ft/s}^2$

Apply Newton's second law.

$F_r = ma_r = (0.12422)(-32)$ $\qquad F_r = -3.98 \text{ lb}$ ◀

$F_\theta = ma_\theta = (0.12422)(32)$ $\qquad F_\theta = 3.98 \text{ lb}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 69.**

Use radial and transverse components of acceleration.

$r = 6(1 + \cos 2\pi t)$ ft $\qquad \theta = 2\pi t$ rad

$\dot{r} = -12\pi \sin 2\pi t$ ft/s $\qquad \dot{\theta} = 2\pi$ rad/s

$\ddot{r} = -24\pi^2 \cos 2\pi t$ ft/s$^2$ $\qquad \ddot{\theta} = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -24\pi^2 \cos 2\pi t - (6 + 6\cos 2\pi t)(2\pi)^2$$

$$= -24\pi^2 - 48\pi^2 \cos 2\pi t \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2)(-12\pi \sin 2\pi t)(2\pi)$$

$$= -48\pi^2 \sin 2\pi t$$

Mass: $m = \dfrac{W}{g} = \dfrac{1}{32.2} = 0.031056$ lb·s$^2$/ft

($a$) $t = 0$. $a_r = -710.61$ ft/s$^2$

$a_\theta = 0$

Apply Newton's second law.

$F_r = ma_r = (0.031056)(710.61)$ $\qquad F_r = -22.1$ lb ◀

$F_\theta = ma_\theta = 0$ $\qquad F_\theta = 0$ ◀

($b$) $t = 0.75$ s. $a_r = -236.87$ ft/s$^2$

$a_\theta = 473.74$ ft/s$^2$

Apply Newton's second law.

$F_r = ma_r = (0.031056)(-236.87)$ $\qquad F_r = -7.36$ lb ◀

$F_\theta = ma_\theta = (0.031056)(473.74)$ $\qquad F_\theta = 14.71$ lb ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 70.

Kinematics: $\dfrac{dr}{dt} = \dot{r} = 1.5$ ft/s, $\quad \ddot{r} = 0$

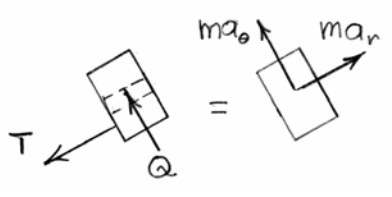
$$\int_0^r dr = \int_0^t r\,dt \qquad \text{or} \qquad r = 1.5t \text{ ft}$$

$$\dot{\theta} = 0.8t \text{ rad/s}, \qquad \ddot{\theta} = 0.8 \text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (1.5t)(0.8t)^2 = -0.96t^3 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1.5t)(0.8) + (2)(1.5)(0.8t) = 3.6t \text{ ft/s}^2$$

Kinetics: Sketch the free body diagrams for the collar.




$$+\nearrow \quad \Sigma F_r = ma_r: \quad -T = ma_r$$

$$+\nwarrow \quad \Sigma F_\theta = ma_\theta: \quad Q = ma_\theta$$

Set $T = Q$ to obtain the required time.

$$-ma_r = ma_\theta \qquad \text{or} \qquad -a_r = a_\theta$$

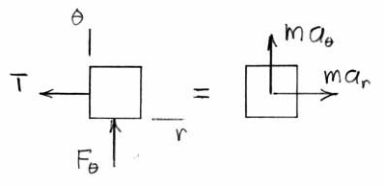
Using the calculated expressions

$$0.96t^3 = 3.6t, \qquad t^2 = \frac{3.6}{0.96} = 3.75 \text{ s}^2$$

$$t = 1.936 \text{ s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 71.




$$\dot{\theta} = 10t \text{ rad/s}, \quad \ddot{\theta} = 10 \text{ rad/s}^2$$

$$m = 0.5/32.2 = 0.015528 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Before cable breaks: $F_r = -T$ and $\ddot{r} = 0$.

$$F_r = ma_r: \quad -T = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$mr\dot{\theta}^2 = m\ddot{r} + T \quad \text{or} \quad \dot{\theta}^2 = \frac{m\ddot{r} + T}{mr} = \frac{0 + 4}{(0.015528)(1.5)} = 171.733 \text{ rad}^2/\text{s}^2$$

$$\dot{\theta} = 13.105 \text{ rad/s}$$

Immediately after the cable breaks: $F_r = 0, \ \dot{r} = 0$

(*a*) Acceleration of *B* relative to the rod.

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = 0 \quad \text{or} \quad \ddot{r} = r\dot{\theta}^2 = (1.5)(13.105)^2 = 257.6 \text{ ft/s}^2$$

$$\mathbf{a}_{B/\text{rod}} = 258 \text{ ft/s}^2 \text{ radially outward} \blacktriangleleft$$

(*b*) Transverse component of the force.

$$F_\theta = ma_\theta: \quad F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

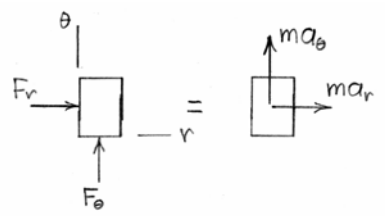
$$(0.015528)\left[(1.5)(10) + (2)(0)(12)\right] = 0.233 \qquad F_\theta = 0.233 \text{ lb} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 72.**

$\dot{\theta} = 15$ rad/s, $m = 230$ g $= 0.230$ kg, $\ddot{\theta} = 0$, $F_\theta = 9$ N, $\ddot{r} = -12$ m/s$^2$

Due to the spring, $F_r = -kr$, $k = 60$ N/m




$$\pm \Sigma F_r = F_r = ma_r: \quad -kr = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\left(k - m\dot{\theta}^2\right)r = -m\ddot{r}$$

(*a*) Radial coordinate.

$$r = -\frac{m\ddot{r}}{k - m\dot{\theta}^2} = -\frac{(0.230)(-12)}{60 - (0.230)(15)^2}$$

$$= 0.33455 \text{ m} \qquad r = 335 \text{ mm}$$

$$\Sigma F_\theta = ma_\theta: \quad F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$2\dot{r}\dot{\theta} = \frac{F_\theta}{m} - r\ddot{\theta}$$

$$\dot{r} = \frac{F_\theta - mr\ddot{\theta}}{2m\dot{\theta}} = \frac{9 - 0}{(2)(0.230)(15)} = 1.304 \text{ m/s}$$

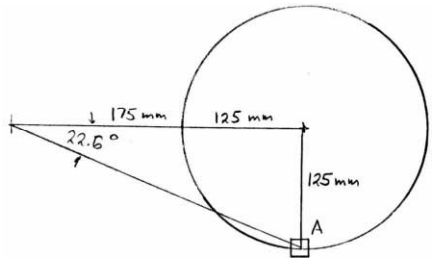
(*b*) Radial component of velocity. $v_r = \dot{r}$ $\qquad v_r = 1.304$ m/s ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 73.

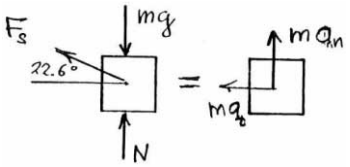
At point $A$. $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{(3.8)^2}{0.125} = 115.52 \text{ m/s}^2 \uparrow$

$$\tan\theta = -\frac{125}{175+125} \qquad \theta = -22.62°$$



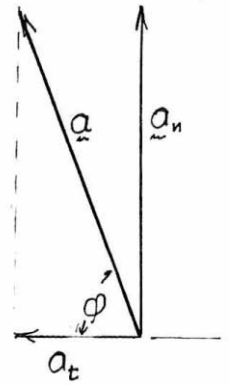

Draw the free body diagram of the collar.

$+\Sigma F = ma$: $\quad F_s \cos 22.6° = ma_t$




$$a_t = \frac{F_s \cos 22.6°}{m} = \frac{70\cos 22.62°}{1.5} = 43.077 \text{ m/s}^2 \leftarrow$$

Acceleration vector.




$$a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{115.52^2 + 43.077^2} = 123.29 \text{ m/s}^2$$

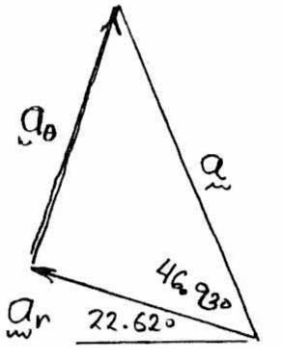
$$\tan\phi = \frac{115.52}{43.077} \qquad \phi = 69.55°$$

$$\phi - 22.62° = 46.93°$$

$$a_r = a\cos 46.93°$$

$$= 123.29\cos 46.93° = 84.2 \text{ m/s}^2 \text{ in negative } r\text{-direction}$$

$$a_r = -84.2 \text{ m/s}^2 \blacktriangleleft$$



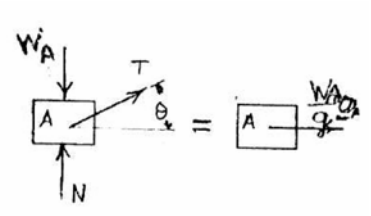

$$a_\theta = a\sin 46.93°$$

$$= 123.29\sin 46.93°$$

$$a_\theta = 90.1 \text{ m/s}^2 \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 74.

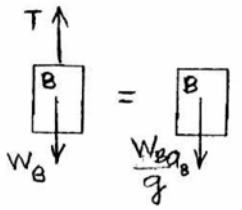



Let $r$ and $\theta$ be polar coordinates of block $A$ as shown, and let $y_B$ be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block $B$.

Constraint of cable: $r + y_B = \text{constant}$,

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \qquad \text{or} \qquad \ddot{r} = -a_B \quad (1)$$

For block $A$, $\xrightarrow{+}\Sigma F_x = m_A a_A$: $T\cos\theta = \dfrac{W_A}{g}a_A$ or $T = \dfrac{W_A}{g}a_A \sec\theta$ (2)

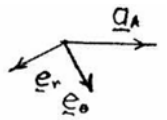



For block $B$, $+\downarrow \Sigma F_y = \dfrac{W_B}{g}a_B$: $W_B - T = \dfrac{W_B}{g}a_B$ (3)

Adding Eq. (1) to Eq. (2) to eliminate $T$, $W_B = \dfrac{W_A}{g}a_A \sec\theta + \dfrac{W_B}{g}a_B$ (4)

Radial and transverse components of $\mathbf{a}_A$.

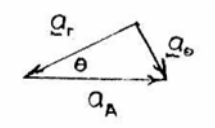
Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.




$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A\cos\theta \quad (5)$$

Noting that initially $\dot{\theta} = 0$, using Eq. (1) to eliminate $\ddot{r}$, and changing signs gives

$$a_B = a_A\cos\theta \quad (6)$$




Substituting Eq. (6) into Eq. (4) and solving for $a_A$,

$$a_A = \frac{W_B g}{W_A \sec\theta + W_B\cos\theta} = \frac{(50)(32.2)}{40\sec 30° + 50\cos 30°} = 17.991 \text{ ft/s}^2$$

From Eq. (6), $a_B = 17.991\cos 30° = 15.581 \text{ ft/s}^2$

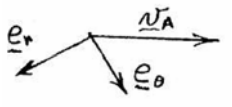
(*a*) From Eq. (2), $T = (40/32.2)(17.991)\sec 30° = 25.81$ $T = 25.8$ lb ◀

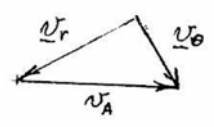
(*b*) Acceleration of block $A$. $\mathbf{a}_A = 17.99 \text{ ft/s}^2 \rightarrow$ ◀

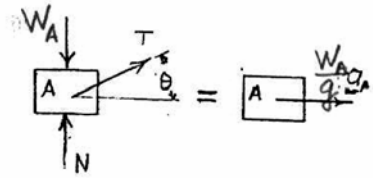
(*c*) Acceleration of block $B$. $\mathbf{a}_B = 15.58 \text{ ft/s}^2 \downarrow$ ◀

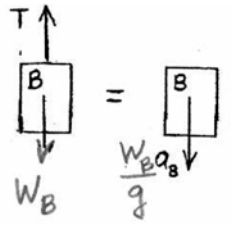

© 2007 The McGraw-Hill Companies.

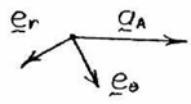
## Chapter 12, Solution 75.

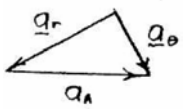












Let $r$ and $\theta$ be polar coordinates of block $A$ as shown, and let $y_B$ be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block $B$.

Radial and transverse components of $\mathbf{v}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.

$$\dot{r} = v_r = \mathbf{v}_A \cdot \mathbf{e}_r = -v_A \cos 30°$$
$$= -6\cos 30° = -5.19615 \text{ ft/s}$$
$$r\dot{\theta} = v_\theta = \mathbf{v}_A \cdot \mathbf{e}_\theta = -v_A \sin 30°$$
$$= 6\sin 30° = 3 \text{ ft/s}$$
$$\dot{\theta} = \frac{v_\theta}{r} = \frac{3}{2.4} = 1.25 \text{ rad/s}$$

Constraint of cable: $r + y_B = \text{constant}$,

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \qquad \text{or} \qquad \ddot{r} = -a_B \quad (1)$$

For block $A$, $\rightarrow \Sigma F_x = m_A a_A : T\cos\theta = \frac{W_A}{g} a_A$ or $T = \frac{W_A}{g} a_A \sec\theta$ (2)

For block $B$, $+\downarrow \Sigma F_y = \frac{W_B}{g} a_B : W_B - T = \frac{W_B}{g} a_B$ (3)

Adding Eq. (1) to Eq. (2) to eliminate $T$, $W_B = \frac{W_A}{g} a_A \sec\theta + \frac{W_B}{g} a_B$ (4)

Radial and transverse components of $\mathbf{a}_A$.

Use a method similar to that used for the components of velocity.

$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \quad (5)$$

Using Eq. (1) to eliminate $\ddot{r}$ and changing signs gives

$$a_B = a_A \cos\theta - r\dot{\theta}^2 \quad (6)$$


© 2007 The McGraw-Hill Companies.

Substituting Eq. (6) into Eq. (4) and solving for $a_A$,

$$a_A = \frac{W_B\left(g + r\dot{\theta}^2\right)}{W_A \sec\theta + W_B \cos\theta} = \frac{(50)\left[32.2 + (2.4)(1.25)^2\right]}{40\sec 30° + 50\cos 30°} = 20.086 \text{ ft/s}^2$$

From Eq. (6), $a_B = 20.086\cos 30° - (2.4)(1.25)^2 = 13.645 \text{ ft/s}^2$

(*a*) From Eq. (2), $T = (40/32.2)(20.086)\sec 30° = 28.8$ $T = 28.8$ lb ◀

(*b*) Acceleration of block *A*. $\mathbf{a}_A = 20.1 \text{ ft/s}^2 \rightarrow$ ◀

(*c*) Acceleration of block *B*. $\mathbf{a}_B = 13.65 \text{ ft/s}^2 \downarrow$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 76.

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27), $h = r^2\dot{\theta} = h_0 = r_0 v_0$

$$\text{or} \quad \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0 \cos 2\theta}{r_0^{\,2}} = \frac{v_0}{r_0}\cos 2\theta$$

Radial component of velocity.

$$v_r = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta}$$

$$= r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}} \frac{v_0}{r}\cos 2\theta \qquad v_r = v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \blacktriangleleft$$

Transverse component of velocity.

$$v_\theta = \frac{h}{r} = \frac{r_0 v_0}{r_0}\sqrt{\cos 2\theta} \qquad v_\theta = v_0\sqrt{\cos 2\theta} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 77.

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27), $h = r^2\dot{\theta} = h_0 = r_0 v_0$ or $\dot{\theta} = \dfrac{r_0 v_n}{r^2} = \dfrac{r_0 v_0 \cos 2\theta}{r_0^2} = \dfrac{v_0}{r_0}\cos 2\theta$

Differentiating the expression for $r$ with respect to time,

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\frac{v_0}{r_0}\cos 2\theta = v_0\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

Differentiating again,

$$\ddot{r} = \frac{d\dot{r}}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(v_0\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = v_0\frac{2\cos^2 2\theta + \sin^2 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = \frac{v_0^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}}$$

(*a*) $v_r = \dot{r} = v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}} = \dfrac{v_0 r}{r_0}\sin 2\theta \qquad v_\theta = r\dot{\theta} = \dfrac{v_0 r}{r_0}\cos 2\theta$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2} = \frac{v_0 r}{r_0}\sqrt{\sin^2 2\theta + \cos^2 2\theta} \qquad v = \frac{v_0 r}{r_0} \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{v_0^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} - \frac{r_0}{\sqrt{\cos 2\theta}}\frac{v_0^2}{r_0^2}\cos^2 2\theta$$

$$= \frac{v_0^2}{r_0}\frac{\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} = \frac{v_0}{r_0\sqrt{\cos 2\theta}} = \frac{v_0^2 r}{r_0^2}$$

$$F_r = ma_r = \frac{mv_0^2 r}{r_0^2}: \qquad F_r = \frac{mv_0^2 r}{r_0^2} \blacktriangleleft$$

Since the particle moves under a central force, $a_\theta = 0$

*continued*


© 2007 The McGraw-Hill Companies.

Magnitude of acceleration: $a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \dfrac{v_0^{\,2} r}{r_0^{\,2}}$

Tangential component of acceleration. $a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(\dfrac{v_0 r}{r_0}\right) = \dfrac{v_0}{r_0}\dot{r} = \dfrac{v_0^{\,2} r}{r_0^{\,2}} \sin 2\theta$

Normal component of acceleration. $a_t = \sqrt{a^2 - a_t^{\,2}} = \dfrac{v_0^{\,2} r}{r_0^{\,2}} \sqrt{1 - \sin^2 2\theta} = \dfrac{v_0^{\,2} r \cos 2\theta}{r_0^{\,2}}$

But $\cos 2\theta = \left(\dfrac{r_0}{r}\right)^2$ Hence, $a_n = \dfrac{v_0^{\,2}}{r}$

(*b*) But $a_n = \dfrac{v^2}{\rho}$ or $\rho = \dfrac{v^2}{a_n} = \dfrac{v_0^{\,2} r^2}{r_0^{\,2}} \cdot \dfrac{r}{v_0^{\,2}}$ $\rho = \dfrac{r^3}{r_0^{\,2}}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 78.

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27), $h = r^2\dot{\theta} = h_0 = r_0 v_0$

$$\text{or} \quad \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{r_0^2 \cos^2\theta} = \frac{v_0}{r_0 \cos^2\theta}$$

Radial component of velocity. $v_r = \dot{r} = \frac{d}{dt}\left(r_0 \cos\theta\right) = -\left(r_0 \sin\theta\right)\dot{\theta}$

Transverse component of velocity. $v_\theta = r\dot{\theta} = \left(r_0 \cos\theta\right)\dot{\theta}$

Speed. $v = \sqrt{v_r^2 + v_\theta^2} = r_0\dot{\theta} = \frac{r_0 v_0}{r_0 \cos^2\theta}$

$$v = \frac{v_0}{\cos^2\theta} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 79.**

Since the particle moves under a central force, $h = \text{constant}$

Using Eq. (12.27), $h = r^2\dot{\theta} = h_0 = r_0 v_0 \qquad \dot{\theta} = \dfrac{r_0 v_0}{r^2} = \dfrac{r_0 v_0}{r_0^2 \cos^2\theta} = \dfrac{v_0}{r_0 \cos^2\theta}$

Radial component of velocity. $v_r = \dot{r} = \dfrac{d}{dt}\left(r_o \cos\theta\right) = -\left(r_0 \sin\theta\right)\dot{\theta}$

Transverse component of velocity. $v_\theta = r\dot{\theta} = \left(r_0 \cos\theta\right)\dot{\theta}$

Speed. $v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = r_0\dot{\theta} = \dfrac{r_0 v_0}{r_0 \cos^2\theta} = \dfrac{v_0}{\cos^2\theta}$

Tangential component of acceleration.

$$a_t = \frac{dv}{dt} = v_0 \frac{(-2)(-\sin\theta)\dot{\theta}}{\cos^3\theta} = \frac{2v_0 \sin\theta}{\cos^3\theta} \cdot \frac{v_0}{r_0 \cos^2\theta} = \frac{2v_0^{\,2} \sin\theta}{r_0 \cos^5\theta}$$

Tangential component of force.

$$F_t = ma_t: \quad F_t = \frac{2mv_0^{\,2} \sin\theta}{r_0 \cos^5\theta}$$

(*a*) $\theta = 0, \quad F_t = 0$ $\qquad F_t = 0$ ◀

(*b*) $\theta = 45°, \quad F_t = \dfrac{2mv_0 \sin 45°}{\cos^5 45°}$ $\qquad F_t = \dfrac{8mv_0^{\,2}}{r_0}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 80.

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

Solving for $\tau$, $\quad \tau = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$

But, $\quad M = \dfrac{4}{3}\pi R^3\rho, \quad$ hence $\quad \sqrt{GM} = 2\sqrt{\dfrac{\pi}{3}G\rho}\, R^{3/2}$

Then, $\quad \tau = \sqrt{\dfrac{3\pi}{G\rho}}\left(\dfrac{r}{R}\right)^{3/2}$

Using $r = 3R$ as given leads to

$$\tau = 3^{3/2}\sqrt{\frac{3\pi}{G\rho}} = 9\sqrt{\frac{\pi}{G\rho}} \qquad \tau = 9\left(\pi/G\rho\right)^{1/2} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 81.

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the period time to complete one orbit.

But $v\tau = 2\pi r$ or $v^2\tau^2 = \dfrac{GM\tau^2}{r} = 4\pi^2 r^2$

Then $r^3 = \dfrac{GM\tau^2}{4\pi^2}$ or $r = \left(\dfrac{GM\tau^2}{4\pi^2}\right)^{1/3}$

Data: $\tau = 23.934 \text{ h} = 86.1624 \times 10^3 \text{ s}$

(*a*) In SI units: $g = 9.81 \text{ m/s}^2$, $R = 6.37 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$r = \left[\frac{\left(398.06 \times 10^{12}\right)\left(86.1624 \times 10^3\right)^2}{4\pi^2}\right]^{1/3} = 42.145 \times 10^6 \text{ m}$$

$$\text{altitude } h = r - R = 35.775 \times 10^6 \qquad h = 35800 \text{ km} \blacktriangleleft$$

In US units: $g = 32.2 \text{ ft/s}^2$, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

$$r = \left[\frac{\left(14.077 \times 10^{15}\right)\left(86.1624 \times 10^3\right)^2}{4\pi^2}\right]^{1/3} = 138.334 \times 10^6 \text{ ft}$$

$$\text{altitude } h = r - R = 117.425 \times 10^6 \text{ ft} \qquad h = 22200 \text{ mi} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

(*b*) In SI units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{398.06 \times 10^{12}}{42.145 \times 10^{6}}} = 3.07 \times 10^{3} \text{ m/s} \qquad v = 3.07 \text{ km/s} \blacktriangleleft$$

In US units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{14.077 \times 10^{15}}{138.334 \times 10^{6}}} = 10.09 \times 10^{3} \text{ ft/s} \qquad v = 10.09 \times 10^{3} \text{ ft/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 82.**

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

from which $GM = \dfrac{4\pi^2 r^3}{\tau^2}$

But $GM = gR^2$, hence, $g = \dfrac{4\pi^2 r^3}{R^2\tau^2}$ (1)

Solving for $r$, $r = \left(\dfrac{gR^2\tau^2}{4\pi^2}\right)^{1/3}$ ◀

Data: $r = 417{,}000 \text{ mi} = 2.202 \times 10^9 \text{ ft}$

$R = 44{,}400 \text{ mi} = 234.4 \times 10^6 \text{ ft}$

$\tau = 3.551 \text{ days} = 85.224 \text{ h} = 306.8 \times 10^3 \text{ s}$

Using (1),

$$g = \frac{4\pi^2\left(2.202 \times 10^9\right)^3}{\left(306.8 \times 10^3\right)^2\left(234.4 \times 10^6\right)^2} = 81.5 \text{ ft/s}^2$$

$g = 81.5 \text{ ft/s}^2$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 83.

Let $M$ be the mass of the sun and $m$ the mass of Venus.

For the circular orbit of Venus,

$$\frac{GMm}{r^2} = ma_n = \frac{mv^2}{r} \qquad GM = rv^2$$

where $r$ is radius of the orbit.

Data: $r = 67.2 \times 10^6 \text{ mi} = 354.8 \times 10^9 \text{ ft}$

$$v = 78.3 \times 10^3 \text{ mi/h} = 114.84 \times 10^3 \text{ ft/s}$$

$$GM = \left(354.8 \times 10^9\right)\left(114.84 \times 10^3\right)^2 = 4.6792 \times 10^{21} \text{ ft}^3/\text{s}^2$$

(*a*) Mass of sun. $$M = \frac{GM}{G} = \frac{4.6792 \times 10^{21} \text{ ft}^3/\text{s}^2}{34.4 \times 10^{-9} \text{ ft}^4/\text{lb}\cdot\text{s}^4}$$

$$M = 136.0 \times 10^{27} \text{ lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

(*b*) At the surface of the sun, $R = 432 \times 10^3 \text{ mi} = 22.81 \times 10^9 \text{ ft}$

$$\frac{GMm}{R^2} = mg$$

$$g = \frac{GM}{R^2} = \frac{4.6792 \times 10^{21}}{\left(22.81 \times 10^9\right)^2} \qquad g = 899 \text{ ft/s}^2 \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 84.

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

But $v = \dfrac{2\pi r}{\tau}$, hence, $\dfrac{2\pi r}{\tau} = \sqrt{\dfrac{GM}{r}}$ or $r^3 = \dfrac{GM\tau^2}{4\pi^2}$

Solving for $r$,

$$r = \left(\frac{GM\tau^2}{4\pi^2}\right)^{1/3} \quad (1)$$

$$v^2 = \frac{GM}{r} = GM\left(\frac{4\pi^2}{GM\tau^2}\right)^{1/3} = \left(\frac{4\pi^2(GM)^2}{\tau^2}\right)^{1/3}$$

$$v = \left(\frac{2\pi GM}{\tau}\right)^{1/3} \quad (2)$$

For earth: $R = 6.37 \times 10^6$ m, $g = 9.81$ m/s$^2$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For Jupiter: $GM = (319)\left(398.06 \times 10^{12}\right) = 126.98 \times 10^{15}$

(*a*) For Ganymede: $\tau = 7.15$ days $= 171.6$ h $= 617.76 \times 10^3$ s

By Eq. (1), $r = \left[\dfrac{\left(126.98 \times 10^{15}\right)\left(617.76 \times 10^3\right)^2}{4\pi^2}\right]^{1/3} = 1.071 \times 10^9$ m $\qquad r = 1.071 \times 10^6$ km ◀

(*b*) For Callisto: $\tau = 16.69$ days $= 400.56$ h $= 1.4420 \times 10^6$ s

By Eq. (2), $v = \left[\dfrac{2\pi\left(126.98 \times 10^{12}\right)}{1.4420 \times 10^6}\right]^{1/3} = 8.209 \times 10^3$ m/s $\qquad v = 8.21$ km/s ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 85.

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

Solving for $r$, $\quad r = \left(\frac{GM\tau^2}{4\pi^2}\right)^{1/3}$

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$, $g = 9.81 \text{ m/s}^2$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

Data: $\tau = 120 \text{ min} = 7200 \text{ s}$

$$r = \left[\frac{\left(398.06 \times 10^{12}\right)(7200)^2}{4\pi^2}\right]^{1/3} = 8.055 \times 10^6 \text{ m}$$

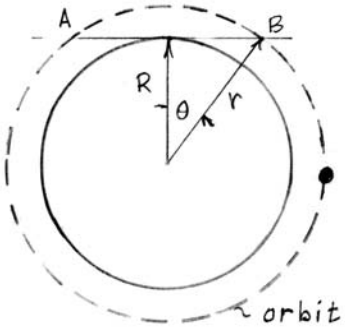
(*a*) altitude $\quad h = r - R = 1.685 \times 10^6 \text{ m}$ $\qquad h = 1685 \text{ km}$ ◀

(*b*) $\quad \cos\theta = \frac{R}{r} = \frac{6.37 \times 10^6}{8.055 \times 10^6} = 0.7908$

$$\theta = 37.74°$$

$$t_{AB} = \frac{2\theta}{360°}\tau = \frac{(75.48)(7200)}{360} = 1509.6 \text{ s}$$

$$t_{AB} = 25.2 \text{ min}$$ ◀




*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 86.

$$\frac{GMm}{r^2} = ma_n \qquad a_n = \frac{GM}{r^2}$$

$$GM = \left(34.4 \times 10^{-9}\ \text{ft}^4/\text{lb}\cdot\text{s}^4\right)\left(5.03 \times 10^{21}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)$$

$$= 173.032 \times 10^{12}\ \text{ft}^3/\text{s}^2$$

(*a*) At the surface of the moon, $a_n = g$

$$r = R = 1080\ \text{mi} = 5.7024 \times 10^6\ \text{ft}$$

$$g = \frac{GM}{R^2} = \frac{\left(34.4 \times 10^{-9}\ \text{ft}^4/\text{lb}\cdot\text{s}^4\right)\left(5.03 \times 10^{21}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)}{\left(5.7024 \times 10^6\,\text{ft}\right)^2}$$

$$g = 5.32\ \text{ft/s}^2 \blacktriangleleft$$

(*b*) Orbit of space vehicle.

$$r = 1080 + 200 = 1280\ \text{mi} = 6.7584 \times 10^6\ \text{ft}$$

$$a_n = \frac{v^2}{r} = \frac{GM}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$\tau = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\tau = \frac{2\pi\left(6.7584 \times 10^6\right)^{3/2}}{\sqrt{173.032 \times 10^{12}}} = 8.3923 \times 10^3\ \text{s}$$

$$\tau = 2.33\ \text{h} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 87.

$$\frac{GMm}{r^2} = ma_n = \frac{mv^2}{r} \qquad v^2 = \frac{GM}{r}$$

$$v = \frac{2\pi r}{\tau} \qquad \frac{4\pi^2 r^2}{\tau^2} = \frac{GM}{r}$$

$$\frac{r^3}{\tau^2} = \frac{GM}{4\pi^2} = \text{constant}$$

Tethys: $r = 183.3 \times 10^3 \text{ mi} = 967.8 \times 10^6 \text{ ft}$

$$\tau = 1.888 \text{ days} = 163.12 \times 10^3 \text{ s}$$

$$\frac{GM}{4\pi^2} = \frac{\left(967.8 \times 10^6\right)^3}{\left(163.12 \times 10^3\right)^2} = 34.068 \times 10^{15} \text{ ft}^3/\text{s}^2$$

Rhea: $\tau = 4.52 \text{ days} = 390.53 \times 10^3 \text{ s}$

$$r^3 = \frac{GM}{4\pi^2}\,\tau^2$$

$$= \left(34.068 \times 10^{15}\right)\left(390.53 \times 10^3\right)^2 = 5.1958 \times 10^{27} \text{ ft}^3$$

(*a*) $r = 1.732 \times 10^9 \text{ ft}$ $\qquad r = 328 \times 10^3 \text{ mi}$ ◀

(*b*) Mass of Saturn.

$$M = \frac{4\pi^2}{G}\left(\frac{GM}{4\pi^2}\right)$$

$$= \frac{4\pi^2\left(34.068 \times 10^{15}\right)}{34.4 \times 10^{-9}} = 39.1 \times 10^{24}$$

$M = 39.1 \times 10^{24} \text{ lb}\cdot\text{s}^2/\text{ft}$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 88.

$$R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

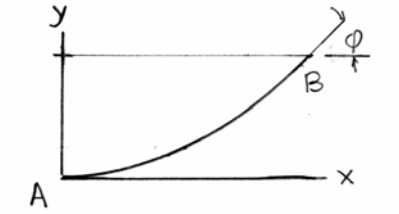
$$r_A = 6370 + 960 = 7330 \text{ km} = 7.33 \times 10^6 \text{ m}$$

$$r_B = 6370 + 8300 = 14670 \text{ km} = 14.67 \times 10^6 \text{ m}$$

$$mr_A v_A = mr_B (v_B)_\theta$$

$$(v_B)_\theta = \frac{r_A v_A}{r_B} = \frac{\left(7.33 \times 10^6\right)\left(10.4 \times 10^3\right)}{14.67 \times 10^6} = 5.196 \times 10^3 \text{ m/s}$$

Parabola $AB$.




$$x_A = 0, \quad y_A = 0$$

$$x_B = r_B = 14.67 \times 10^6 \text{ m}$$

$$y_B = r_A = 7.33 \times 10^6 \text{ m}$$

For a parabolic trajectory, $y = kx^2$ from which $k = \dfrac{y_B}{x_B^2}$

Differentiating with respect to $x$, $\dfrac{dy}{dx} = 2kx = \dfrac{2y_B x}{x_B^2}$

At point $B$, $\dfrac{dy}{dx} = \tan\phi = \dfrac{2y_B}{x_B} = \dfrac{(2)\left(7.33 \times 10^6\right)}{14.67 \times 10^6} = 1.0000$

$$\phi = 45°$$

$$v_B = \frac{(v_B)_\theta}{\sin 45°} = \frac{5.196 \times 10^3}{\sin 45°} = 7.349 \times 10^3 \text{ m/s}$$

$$\boldsymbol{v}_B = 7.35 \times 10^3 \text{ m/s} \angle 45° \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 89.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$ $\quad (GM)_{\text{earth}} = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$

For sun, $(GM)_{\text{sun}} = \left(332.8 \times 10^3\right)\left(14.077 \times 10^{15}\right) = 4.6849 \times 10^{21} \text{ ft}^3/\text{s}^2$

For circular orbit of earth, $r_E = 92.96 \times 10^6 \text{ mi} = 490.8 \times 10^9 \text{ ft}$

$$v_E = \sqrt{\frac{(GM)_{\text{sun}}}{r_E}} = \sqrt{\frac{4.6849 \times 10^{21}}{490.8 \times 10^9}} = 97.70 \times 10^3 \text{ ft/s}$$

For transfer orbit $AB$, $\quad r_A = r_E, \quad r_B = r_M = 141.5 \times 10^6 \text{ mi} = 747.12 \times 10^9 \text{ ft}$

$$v_A = v_E + (\Delta v)_A = 97.70 \times 10^3 + (1.83)(5280) = 107.36 \text{ ft/s}$$

$$mr_A v_A = mr_B v_B$$

$$v_B = \frac{r_A v_A}{r_B} = \frac{\left(490.8 \times 10^9\right)\left(107.36 \times 10^3\right)}{747.12 \times 10^3} = 70.527 \text{ ft/s}$$

For circular orbit of Mars, $r_M = 141.5 \times 10^6 \text{ mi} = 747.12 \times 10^9 \text{ ft}$

$$v_M = \sqrt{\frac{(GM)_{\text{sun}}}{r_M}} = \sqrt{\frac{4.6849 \times 10^{21}}{747.12 \times 10^9}} = 79.187 \times 10^3 \text{ ft/s}$$

Speed increase at $B$.

$$(\Delta v)_B = v_M - v_B = 79.187 \times 10^3 - 70.527 \times 10^3 = 8.660 \times 10^3 \text{ ft/s}$$

$$(\Delta v)_B = 1.640 \text{ mi/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 90.

Circular orbits: $v = \sqrt{\dfrac{GM}{r}}$

$$r_A = 1400 \text{ mi} = 7.392 \times 10^6 \text{ ft}$$

$$(v_A)_1 = \sqrt{\frac{\left(34.4 \times 10^{-9}\right)\left(5.03 \times 10^{21}\right)}{7.392 \times 10^6}} = 4.8382 \times 10^3 \text{ ft/s}$$

$$r_B = 1300 \text{ mi} = 6.864 \times 10^6 \text{ ft}$$

$$(v_B)_2 = \sqrt{\frac{\left(34.4 \times 10^{-9}\right)\left(5.03 \times 10^{21}\right)}{6.864 \times 10^6}} = 5.0208 \times 10^3 \text{ ft/s}$$

(*a*) Transfer orbit *AB*.

$$(v_A)_2 = (v_A)_1 + (\Delta v)_A = 4.8382 \times 10^3 - 86 = 4.7522 \times 10^3 \text{ ft/s}$$

$$mr_A(v_A)_2 = mr_B(v_B)_1$$

$$(v_B)_1 = \frac{r_A(v_A)_2}{r_B} = \frac{\left(7.392 \times 10^6\right)\left(4.7522 \times 10^3\right)}{6.864 \times 10^3} = 5.1178 \times 10^3 \text{ ft/s}$$

$$(v_B)_1 = 5.12 \times 10^3 \text{ ft/s} \blacktriangleleft$$

(*b*) Speed change at *B*.

$$(\Delta v_B) = (v_B)_2 - (v_B)_1 = 5.0208 \times 10^3 \text{ ft/s} - 5.1178 \times 10^3 \text{ ft/s} = -97.0 \text{ ft/s}$$

Speed reduction at *B*. $|\Delta v_B| = 97.0 \text{ ft/s} \blacktriangleleft$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 91.

$$R = 6370 \text{ km} = 6.37 \times 10^6 \text{m}, \quad r_A = r_D = 6370 + 610 = 6980 \text{ km} = 6.98 \times 10^6 \text{m}$$

$$r_B = 6370 + 290 = 6660 \text{ km} = 6.66 \times 10^6 \text{m}, \quad r_C = 6900 \text{ km} = 6.90 \times 10^6 \text{m}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{gR^2}{r_A}} = \sqrt{\frac{(9.81)\left(6.37 \times 10^6\right)^2}{6.98 \times 10^6}} = 7.55173 \times 10^3 \text{ m/s}$$

$$(v_B)_{\text{circ}} = \sqrt{\frac{gR^2}{r_B}} = \sqrt{\frac{(9.81)\left(6.37 \times 10^6\right)^2}{6.66 \times 10^6}} = 7.73102 \times 10^3 \text{ m/s}$$

For path *BC*.

$$v_B = (v_B)_{\text{circ}} + (\Delta v)_B = 7.73102 \times 10^3 + 85 = 7.81602 \times 10^3 \text{ m/s}$$

$$mr_B v_B = mr_C (v_C)_1$$

$$(v_C)_1 = \frac{r_B v_B}{r_C} = \frac{\left(6.66 \times 10^6\right)\left(7.81602 \times 10^3\right)}{6.90 \times 10^6} = 7.54416 \times 10^3 \text{ m/s}$$

For path *CD*.

$$(v_C)_2 = (v_C)_1 + (\Delta v)_C = 7.54416 \times 10^3 + 79 = 7.62316 \times 10^3 \text{ m/s}$$

$$mr_C (v_C)_2 = mr_D v_D$$

$$v_D = \frac{r_C (v_C)_2}{r_D} = \frac{\left(6.90 \times 10^6\right)\left(7.62316 \times 10^3\right)}{6.98 \times 10^6} = 7.53579 \times 10^3 \text{ m/s}$$

$$(\Delta v)_D = (v_A)_{\text{circ}} - v_D = 7.55173 \times 10^3 - 7.53579 \times 10^3 = 15.94 \text{ m/s}$$

$$(\Delta v)_D = 15.94 \text{ m/s} \blacktriangleleft$$

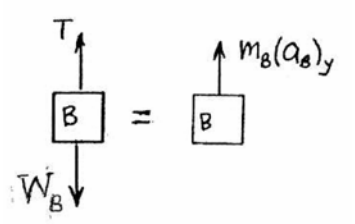

© 2007 The McGraw-Hill Companies.

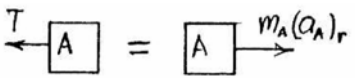
## Chapter 12, Solution 92.

Masses: $m_A = m_B = \dfrac{2.6}{32.2} = 0.08075 \text{ lb}\cdot\text{s}^2/\text{ft}$

Let $y$ be the position coordinate of $B$, positive upward with origin at $O$.

Constraint of the cord: $r - y = \text{constant}$ or $\ddot{y} = \ddot{r}$




(*a*) Kinematics: $(a_B)_y = \ddot{y} = \ddot{r}$ and $(a_A)_r = \ddot{r} - r\dot{\theta}^2$

Collar $B$: $\Sigma F_y = m_B a_B$: $T - W_B = m_B\ddot{y} = m_B\ddot{r}$ (1)

Collar $A$: $\overset{+}{\rightarrow} \Sigma F_r = m_A(a_A)_r$: $-T = m_A\left(\ddot{r} - r\dot{\theta}^2\right)$ (2)

Adding (1) and (2) to eliminate $T$,

$$-W_B = (m_A + m_B)\ddot{r} + m_A r\dot{\theta}^2$$

$$a_{A/\text{rod}} = \ddot{r} = \frac{m_A r\dot{\theta}^2 - W_B}{m_A + m_B} = \frac{(0.08075)(0.6)(10)^2 - (2.6)}{0.08075 + 0.08075}$$

$$\mathbf{a}_{A/\text{rod}} = 13.90 \text{ ft/s}^2 \rightarrow \blacktriangleleft$$

From (1), $T = m_B(\ddot{r} + g) = (0.08075)(13.90 + 32.2)$

$$T = 3.72 \text{ lb} \blacktriangleleft$$

(*b*) Conservation of angular momentum of collar $A$: $(H_0)_2 = (H_0)_1$

$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$

$$(v_\theta)_2 = \frac{r_1(v_\theta)_1}{r_2} = \frac{r_1^2\dot{\theta}}{r_2} = \frac{(0.6)^2(10)}{0.9} = 4.00$$

$$(v_\theta)_2 = 4.00 \text{ ft/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 93.

Masses: $m_A = m_B = \dfrac{2.6}{32.2} = 0.08075 \text{ lb}\cdot\text{s}^2/\text{ft}$

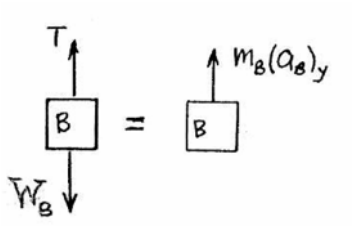
(*a*) Conservation of angular momentum of collar *A*: $(H_0)_2 = (H_0)_1$

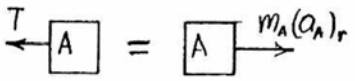
$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$

$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{r_1^2 \dot{\theta}_1}{r_2} = \frac{(0.6)^2 (12)}{1.2} = 3.6$$

$$(v_\theta)_2 = 3.60 \text{ ft/s} \blacktriangleleft$$

$$\dot{\theta}_2 = \frac{(v_\theta)_2}{r_A} = \frac{3.6}{1.2} = 3.00 \text{ rad/s}$$




(*b*) Let *y* be the position coordinate of *B*, positive upward with origin at *O*.

Constraint of the cord: $r - y = \text{constant}$ or $\ddot{y} = \ddot{r}$

Kinematics:

$$(a_B)_y = \ddot{y} = \ddot{r} \qquad \text{and} \qquad (a_A)_r = \ddot{r} - r\dot{\theta}^2$$

Collar *B*: $\Sigma F_y = m_B a_B$: $T - W_B = m_B \ddot{y} = m_B \ddot{r}$ (1)

Collar *A*: $\overset{+}{\rightarrow} \Sigma F_r = m_A (a_A)_r$: $-T = m_A \left(\ddot{r} - r\dot{\theta}^2\right)$ (2)

Adding (1) and (2) to eliminate *T*,

$$-W_B = (m_A + m_B)\ddot{r} + m_A r\dot{\theta}^2$$

$$a_{A/\text{rod}} = \ddot{r} = \frac{m_A r\dot{\theta}^2 - W_B}{m_A + m_B} = \frac{(0.08075)(1.2)(3.00)^2 - (2.6)}{0.08075 + 0.08075} = -10.70 \text{ ft/s}^2$$

$$T = m_B (\ddot{r} + g) = (0.08075)(-10.70 + 32.2) \qquad T = 1.736 \text{ lb} \blacktriangleleft$$

$$a_{A/\text{rod}} = 10.70 \text{ ft/s}^2 \text{ radially inward.} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 94.

Since friction and mass of the rod are neglected, the resultant force acting on the collar is the spring force, which is a central force. The angular momentum of the collar about the shaft is constant.

$$mrv_\theta = mr_0v_0 = mr_0^2\dot{\theta}_0$$

(*a*) $$v_\theta = \frac{r_0^2\dot{\theta}_0}{r} = \frac{(0.150)^2(12)}{0.600}$$ $v_\theta = 0.450$ m/s ◀

Calculate the spring force.

Elongation: $e = 0.600 - 0.750 = -0.150$ m

Force: $F_r = -ke = -(5)(-0.150) = 0.750$ N

Apply Newton's second law.

$$F_r = ma_r$$

(*b*) $$a_r = \frac{F_r}{m} = \frac{0.750}{0.300}$$ $a_r = 2.50$ m/s$^2$ ◀

$$F_\theta = 0$$

$$a_\theta = \frac{F_\theta}{m}$$ $a_\theta = 0$ ◀

(*c*) Acceleration of *B* relative to the rod.

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - \frac{v_\theta^2}{r}$$

$$a_{B/rod} = \ddot{r} = a_r + \frac{v_\theta^2}{r} = 2.50 + \frac{(0.450)^2}{0.600}$$

$a_{B/rod} = 2.84$ m/s$^2$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 95.

Data from Prob. 12.94.

$m = 300$ g $= 0.300$ kg, $k = 5$ N/m, $\dot{\theta}_0 = 12$ rad/s,

$l_0 = 750$ mm $= 0.750$ m, $r_0 = 150$ mm $= 0.150$ m, $r = 600$ mm $= 0.600$ m.

Since friction and mass of the rod are neglected, the resultant force acting on the collar is the spring force, which is a central force. The angular momentum of the collar about the shaft is constant.

$$mrv_\theta = mr^2\dot{\theta} = mr_0^2\dot{\theta}_0$$

$$\dot{\theta} = \frac{r_0^2\dot{\theta}_0}{r^2}$$

(*a*) Radial component $v_r$ of the collar.

Kinematics: $a_r = \ddot{r} - r\dot{\theta}^2$

Newton's second law: $F_r = -k(r - l_0) = ma_r$

Eliminating $a_r$, $\ddot{r} - r\dot{\theta}^2 = -\frac{k}{m}(r - l_0)$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \left(\frac{d\dot{r}}{dr}\right)\dot{r} = \frac{1}{2}\frac{d}{dr}(\dot{r})^2$$

$$\frac{1}{2}\frac{d}{dr}(\dot{r})^2 = r\dot{\theta}^2 - \frac{k}{m}(r - l_0)$$

$$= \frac{r_0^4\dot{\theta}_0^2}{r^3} - \frac{k}{m}(r - l_0)$$

Separate variables and integrate with respect to $r$, noting that $\dot{r} = 0$ when $r = r_0$.

$$d\left(\frac{\dot{r}^2}{2}\right) = \frac{r_0^2\dot{\theta}_0^2}{r}dr - \frac{k}{m}(r - l_0)dr$$

$$\left.\frac{\dot{r}^2}{2}\right|_0^{\dot{r}} = r_0^2\dot{\theta}_0^2\int_{r_0}^{r}\frac{dr}{r^3} - \frac{k}{m}\int_{r_0}^{r}(r - l_0)\,dr$$

*continued*


© 2007 The McGraw-Hill Companies.

$$\frac{1}{2}\dot{r}^2 = r_0^4\dot{\theta}_0^2\left[-\frac{1}{2r^2}\right]_{r_0}^{r} - \frac{k}{m}\left[\frac{1}{2}(r-l_0)^2\right]_{r_0}^{r}$$

$$\dot{r}^2 = r_0^2\dot{\theta}_0^2\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right) - \frac{k}{m}\left[(r-l_0)^2-(r_0-l_0)^2\right]$$

$$= (0.150)^4(12)^2\left(\frac{1}{0.150^2}-\frac{1}{0.600^2}\right) - \frac{5}{0.3}\left[(0.600-0.750)^2-(0.150-0.750)^2\right]$$

$$= 3.0375 + 5.625 = 8.6625 \text{ m}^2/\text{s}^2$$

$$\dot{r} = 2.9432 \text{ m/s} \qquad v_r = 2.94 \text{ m/s} \blacktriangleleft$$

(*b*) Value of $\ddot{\theta}$.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{F_\theta}{m} = 0$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

where $\dot{\theta} = \dfrac{r_0^2\dot{\theta}_0}{r^2} = \dfrac{(0.150)^2(12)}{(0.600)^2} = 0.750$ rad/s

$$\ddot{\theta} = -\frac{(2)(2.9432)(0.750)}{0.600} \qquad \ddot{\theta} = -7.36 \text{ rad/s}^2 \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 96.

$$u = \frac{1}{r} = \frac{2 - \cos\theta}{r_0}, \quad \frac{du}{d\theta} = \frac{\sin\theta}{r_0}, \quad \frac{d^2u}{d\theta^2} = \frac{\cos\theta}{r_0}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{2}{r_0} = \frac{F}{mh^2u^2} \quad \text{by Eq. } (12.37)$$

Solving for $F$,

$$F = \frac{2mh^2u^2}{r_0} = \frac{2mh^2}{r_0 r^2}$$

Since $m$, $h$, and $r_0$ are constants, $F$ is proportional to $\frac{1}{r^2}$, or inversely proportional to $r^2$.

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 97.

$$u = \frac{1}{r} = \frac{6\cos\theta - 5}{r_0}, \quad \frac{du}{d\theta} = -\frac{6\sin\theta}{r_0}, \quad \frac{d^2u}{d\theta} = \frac{6\cos\theta}{r_0}$$

$$\frac{d^2u}{d\theta^2} + u = -\frac{5}{r_0} = \frac{F}{mh^2u^2} \quad \text{by Eq. } (12.37).$$

Solving for $F$,

$$F = -\frac{5mh^2u^2}{r_0} = -\frac{5mh^2}{r_0r^2}$$

Since $m$, $h$, and $r_0$ are constants, $F$ is proportional to $\frac{1}{r^2}$, or inversely proportional to $r^2$. The minus sign indicates that the force is repulsive, as shown in Fig. P12.97.

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 98.

From Fig. P12.98, $x = r\sin\theta, \quad y = r_0 - r\cos\theta$

But $y = \dfrac{x^2}{4r_0}$ or $r_0 - r\cos\theta = \dfrac{r^2\sin^2\theta}{4r_0} = \dfrac{r^2\left(1-\cos^2\theta\right)}{4r_0}$

$$\left(\frac{1-\cos^2\theta}{4r_0}\right)r^2 + \left(\cos\theta\right)r - r_0 = 0$$

Solving the quadratic equation for $r$,

$$r = \frac{2r_0}{1-\cos^2\theta}\left[-\cos\theta \pm \sqrt{\cos^2\theta - 4\left(\frac{1-\cos^2\theta}{4r_0}\right)\left(-r_0\right)}\right]$$

$$= \frac{2r_0}{1-\cos^2\theta}\left(-\cos\theta \pm 1\right) = \frac{2r_0\left(1-\cos\theta\right)}{1-\cos^2\theta}$$

since $r > 0$. Simplifying gives

$$r = \frac{2r_0}{1+\cos\theta} \qquad \text{or} \qquad \frac{1}{r} = \frac{1+\cos\theta}{2r_0} = u \tag{1}$$

$$\frac{du}{d\theta} = -\frac{\sin\theta}{2r_0} \qquad \text{and} \qquad \frac{d^2u}{d\theta^2} = -\frac{\cos\theta}{2r_0}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{1}{2r_0} = \frac{F}{mh^2u^2}$$

Solving for $F$, $$F = \frac{mh^2u^2}{2r_0} = \frac{mh^2}{2r_0r^2}$$

Since $m$, $h$, and $r_0$ are constants, F is proportional to $\dfrac{1}{r^2}$, or inversely proportional to $r^2$.

By Eq. $(12.39')$, $\dfrac{1}{r} = \dfrac{GM}{h^2}\left(1+\varepsilon\cos\theta\right) = u$

Comparing with (1) shows that $\varepsilon = 1$ and

$$\frac{GM}{h^2} = \frac{1}{2r_0} \qquad h = \sqrt{2GMr_0} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 99.

$$u = \frac{1}{r} = \frac{1}{r_0} e^{-b\theta}$$

$$\frac{du}{d\theta} = -\frac{b}{r_0} e^{-b\theta}$$

$$\frac{d^2u}{d\theta^2} = \frac{b^2}{r_0} e^{-b\theta}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{b^2 + 1}{r_0} e^{-b\theta} = \frac{F}{mh^2u^2}$$

$$\mathbf{F} = \frac{(b^2 + 1)mh^2u^2}{r_0} e^{-b\theta}$$

$$= \frac{(b^2 + 1)mh^2u^2}{r} = \frac{(b^2 + 1)mh^2}{r^3}$$

Since $b$, $m$, and $h$ are constants, $\mathbf{F}$ is proportional to $\frac{1}{r^3}$, or inversely proportional to $r^3$.


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 100.

For a parabolic trajectory, the velocity $v_0$ at the perigee is equal to the escape velocity.

$$v_0 = v_{esc} = \sqrt{\frac{2GM}{r_0}}$$

Solving for $M$,

$$M = \frac{r_0 v_0^2}{2G}$$

Data:

$$r_0 = 350 \times 10^3 \text{ km} = 350 \times 10^6 \text{ m}$$

$$v_0 = 26.9 \text{ km/s} = 26.9 \times 10^3 \text{ m/s}$$

$$G = 66.73 \times 10^{-12} \text{ m}^3/\text{kg}\cdot\text{s}^2$$

$$M = \frac{(350 \times 10^6)(26.9 \times 10^3)^2}{(2)(66.73 \times 10^{-12})}$$

$$M = 1.898 \times 10^{27} \text{ kg} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 101.

$$GM = \left(34.4 \times 10^{-9}\ \text{ft}^4/\text{lb}\cdot\text{s}^4\right)\left(334 \times 10^{21}\ \text{lb}\cdot\text{s}^2/\text{ft}\right) = 11.490 \times 10^{15}\ \text{ft}^3/\text{s}^2$$

$$R = 3761\ \text{mi} = 19.858 \times 10^6\ \text{ft}$$

$$r_0 = 3761 + 175 = 3936\ \text{mi} = 20.782 \times 10^6\ \text{ft}$$

(*a*) Velocity of probe as it approaches *A*.

$$v_0 = \sqrt{\frac{2GM}{r_0}} = \sqrt{\frac{(2)\left(11.490 \times 10^{15}\right)}{20.782 \times 10^6}} = 33.252 \times 10^3\ \text{ft/s}$$

or $v_0 = 6.30\ \text{mi/s}$ ◀

$$v_{\text{cir}} = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{11.490 \times 10^{15}}{20.782 \times 10^6}} = 23.513 \times 10^3\ \text{ft/s}$$

(*b*) Decrease velocity at *A*.

$$\Delta v = v_{\text{circ}} - v_0 = -9.739 \times 10^3\ \text{ft/s} = -1.845\ \text{mi/s}$$

or $|\Delta v| = 1.845\ \text{mi/s}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 102.

For earth, $R = 6.37 \times 10^6$ m $\qquad g = 9.81$ m/s$^2$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For Io, $GM = (0.01496)\left(398.06 \times 10^{12}\right) = 5.955 \times 10^{12}$ m$^3$/s$^2$

$$r_0 = 2.820 \times 10^6 \text{ m}, \qquad v_0 = 15 \times 10^3 \text{ m/s}$$

$$h = r_0 v_0 = 42.3 \times 10^9 \text{ m}^2/\text{s}$$

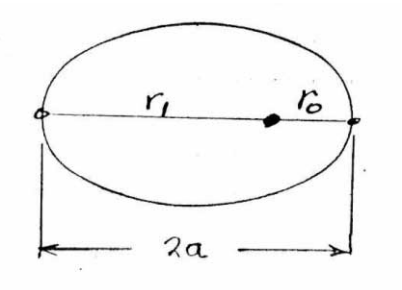
$$\frac{1}{r} = \frac{GM}{h^2}(1 + \varepsilon \cos\theta) \qquad \text{and} \qquad \frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon)$$

$$1 + \varepsilon = \frac{h^2}{r_0 GM} = \frac{\left(42.3 \times 10^9\right)^2}{\left(2.820 \times 10^6\right)\left(5.955 \times 10^{12}\right)} = 106.5$$

$$\varepsilon = 105.5 \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

**Chapter 12, Solution 103.**




Earth: $R = 6.37 \times 10^6$ m $\qquad g = 9.81$ m/s$^2$

$$GM_e = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

Jupiter: $GM = (318)(398.06 \times 10^{12}) = 126.583 \times 10^{15}$ m$^3$/s$^2$

Orbit of satellite. $a = 23.6 \times 10^9$ m

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \qquad \frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon)$$

$$r_0 = \frac{h^2}{GM(1+\varepsilon)} \qquad r_1 = \frac{h^2}{GM(1-\varepsilon)}$$

$$2a = r_0 + r_1 = \frac{2h^2}{GM(1-\varepsilon^2)}$$

$$h^2 = (1-\varepsilon^2)GMa = (1-0.45^2)(126.583 \times 10^{15})(23.6 \times 10^9)$$

$$= 2.3824 \times 10^{27} \text{ m}^4/\text{s}^2$$

$$h = 48.810 \times 10^{12} \text{ m/s}$$

$$r_0 = \frac{2.3824 \times 10^{27}}{(126.583 \times 10^{15})(1+0.45)} = 12.980 \times 10^9 \text{ m}$$

$$r_1 = \frac{2.3824 \times 10^{27}}{(126.583 \times 10^{15})(1-0.45)} = 34.220 \times 10^9 \text{ m}$$

$$v_{\max} = \frac{h}{r_0} = \frac{48.810 \times 10^{12}}{12.980 \times 10^9} = 3.760 \times 10^3 \qquad v_{\max} = 3760 \text{ m/s} \blacktriangleleft$$

$$v_{\min} = \frac{h}{r_1} = \frac{48.810 \times 10^{12}}{34.220 \times 10^9} = 1.426 \times 10^3 \qquad v_{\min} = 1426 \text{ m/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 104.

Using Eq. $(12.39)$, $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B.$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{1}{r_o} + \dfrac{1}{r_1} = \dfrac{2GM}{h^2}$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 105.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$ $\quad GM = gR^2 = (32.2)\left(20.909 \times 10^6\right) = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$

$$r_A = 3960 + 11870 = 15830 \text{ mi} = 83.582 \times 10^6 \text{ ft}$$

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077 \times 10^{15}}{83.582 \times 10^6}} = 12.978 \times 10^3 \text{ ft/s}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r_A}} = \sqrt{2}\, v_{\text{circ}} = 18.353 \times 10^3 \text{ ft/s}$$

(*a*) Increase in speed at *A*.

$$\Delta v = v_{\text{esc}} - v_{\text{circ}} = 5.375 \times 10^3 \text{ ft/s} \qquad \Delta v = 5.38 \times 10^3 \text{ ft/s} \blacktriangleleft$$

Elliptical orbit with $\quad r_B = 3960 + 3960 = 7920 \text{ mi} = 41.818 \times 10^6 \text{ ft}.$

Using Eq. (12.37), $\quad \dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A \quad$ and $\quad \dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\quad \theta_B = \theta_A + 180°, \;$ so that $\; \cos\theta_A = -\cos\theta_B$

Adding, $\quad \dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(14.077 \times 10^{15}\right)\left(83.582 \times 10^6\right)\left(41.818 \times 10^6\right)}{125.4 \times 10^6}}$$

$$= 885.848 \times 10^9 \text{ ft}^2/\text{s}$$

$$v_A = \frac{h}{r_A} = \frac{885.848 \times 10^9}{83.582 \times 10^6} = 10.599 \times 10^3 \text{ ft/s}$$

(*b*) Decrease in speed. $\quad \Delta v = v_{\text{circ}} - v_A = 2.379 \times 10^3 \text{ ft/s} \qquad \Delta v = 2380 \text{ ft/s} \blacktriangleleft$

(*c*) $\quad \dfrac{1}{r_B} - \dfrac{1}{r_A} = \dfrac{r_A - r_B}{r_A r_B} = C\cos\theta_B - C\cos\theta_A = 2C$

$$C = \frac{r_A - r_B}{2r_Ar_B} = \frac{41.764 \times 10^6}{(2)\left(83.582 \times 10^6\right)\left(41.818 \times 10^6\right)} = 5.974 \times 10^{-9} \text{ ft}^{-1}$$

By Eq. (12.40), $\quad \varepsilon = \dfrac{Ch^2}{GM} = \dfrac{\left(5.974 \times 10^{-9}\right)\left(885.848 \times 10^9\right)^2}{14.077 \times 10^{15}}$

$$\varepsilon = 0.333 \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 106.

For earth, $R = 6370 \text{ km} = 6.370 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$r_A = 6370 + 10,000 = 16370 \text{ km} = 16.370 \times 10^6 \text{ m}$$

$$r_B = 6370 + 140,000 = 146,370 = 146.37 \times 10^6 \text{ m}$$

For a circular orbit with $r_0 = r_A$,

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{398.06 \times 10^{12}}{16.370 \times 10^6}} = 4.931 \times 10^3 \text{ m/s}$$

Elliptic orbit.

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_B = -\cos\theta_A$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(398.06 \times 10^{12}\right)\left(16.370 \times 10^6\right)\left(146.37 \times 10^6\right)}{162.74 \times 10^6}}$$

$$= 108.27 \times 10^9 \text{ m}^2/\text{s}$$

$$v_A = \frac{h}{r_A} = \frac{108.27 \times 10^9}{16.370 \times 10^6} = 6.614 \times 10^3 \text{ m/s}$$

(*a*) Increase in speed at *A*.

$$\Delta v_A = v_A - v_{\text{circ}} = 1.683 \times 10^3 \text{ m/s} \qquad \Delta v_A = 1.683 \times 10^3 \text{ m/s} \blacktriangleleft$$

(*b*) Speed of observatory at *B*.

$$v_B = \frac{h}{r_B} = \frac{108.27 \times 10^9}{146.37 \times 10^6} \qquad v_B = 7.40 \times 10^3 \text{ m/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 107.

For earth, $R = 6.37 \times 10^6$ m

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For sun: $GM = \left(332.8 \times 10^3\right)\left(398.06 \times 10^{12}\right) = 132.474 \times 10^{18} \text{ m}^3/\text{s}^2$

For elliptic orbit $AB$, $r_A = 325 \times 10^9$ m, $r_B = 148 \times 10^9$ m

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_A = -\cos\theta_B$.

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h_{AB}^2}$

$$h_{AB} = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(132.474 \times 10^{18}\right)\left(325 \times 10^9\right)\left(148 \times 10^9\right)}{473 \times 10^9}}$$

$$= 5.1907 \times 10^{15} \text{ m}^2/\text{s}$$

(*a*) $$\left(v_A\right)_1 = \frac{h_{AB}}{r_A} = \frac{5.1907 \times 10^{15}}{325 \times 10^9} = 15.971 \times 10^3 \qquad \left(v_A\right)_1 = 15.97 \times 10^3 \text{ m/s} \blacktriangleleft$$

For transfer orbit $AB'$, $r_{B'} = 137.6 \times 10^9$ m

*continued*


© 2007 The McGraw-Hill Companies.

$$h_{AB'} = \sqrt{\frac{2GMr_A r_{B'}}{r_A + r_{B'}}} = \sqrt{\frac{(2)\left(132.474 \times 10^{18}\right)\left(325 \times 10^{9}\right)\left(137.6 \times 10^{9}\right)}{462.6 \times 10^{9}}}$$

$$= 5.0609 \times 10^{15} \text{ m}^2/\text{s}$$

$$\left(v_A\right)_2 = \frac{h_{AB'}}{r_A} = \frac{5.0609 \times 10^{15}}{325 \times 10^{9}} = 15.572 \times 10^{3} \text{ m/s}$$

(*b*) Decrease of speed at *A*.

$$\left(\Delta v_A\right) = \left(v_A\right)_1 - \left(v_A\right)_2 = 399 \text{ m/s} \qquad \left|\Delta v_A\right| = 399 \text{ m/s} \blacktriangleleft$$

$$\left(v_B\right)_1 = \frac{h_{AB'}}{r_{B'}} = \frac{5.0609 \times 10^{15}}{137.6 \times 10^{9}} = 36.780 \times 10^{3} \text{ m/s}$$

For elliptic orbit $B'A'$, $r_{A'} = 264.7 \times 10^{9}$ m

$$h_{B'A'} = \sqrt{\frac{2GMr_{B'} r_{A'}}{r_{B'} + r_{A'}}} = \sqrt{\frac{(2)\left(132.474 \times 10^{18}\right)\left(137.6 \times 10^{9}\right)\left(264.7 \times 10^{9}\right)}{402.3 \times 10^{9}}}$$

$$= 4.8977 \times 10^{15} \text{ m}^2/\text{s}$$

$$\left(v_{B'}\right)_2 = \frac{h_{B'A'}}{r_{B'}} = \frac{4.8977 \times 10^{15}}{137.6 \times 10^{9}} = 35.594 \times 10^{3} \text{m/s}$$

Decrease in speed at $B'$:

$$\left(v_{B'}\right)_1 - \left(v_{B'}\right)_2 = 1.186 \times 10^{3} \qquad \left|\Delta v_B\right| = 1.186 \times 10^{3} \text{ m/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 108.

For Earth, $GM_{\text{earth}} = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12}\ \text{m}^3/\text{s}^2$

For Venus, $GM = 0.82\ \ GM_{\text{earth}} = 326.41 \times 10^{12}\ \text{m}^3/\text{s}^2$

For a parabolic trajectory with $r_A = 15 \times 10^6\ \text{m}$

$$\left(v_A\right)_1 = v_{\text{esc}} = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)\left(326.41 \times 10^{12}\right)}{15 \times 10^6}} = 6.5971 \times 10^3\ \text{m/s}$$

First transfer orbit *AB*. $r_B = 300 \times 10^6\ \text{m}$

At point *A*, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C\cos 180° = \frac{GM}{h_{AB}^2} - C \tag{1}$$

At point *B*, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C\cos 0 = \frac{GM}{h_{AB}^2} + C \tag{2}$$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_B + r_A}{r_A r_B} = \dfrac{2GM}{h_{AB}^2}$

Solving for $h_{AB}$,

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_B + r_A}} = \sqrt{\frac{(2)\left(326.41 \times 10^{12}\right)\left(15 \times 10^6\right)\left(300 \times 10^6\right)}{315 \times 10^6}}$$

$$= 96.571 \times 10^9\ \text{m}^2/\text{s}$$

*continued*


© 2007 The McGraw-Hill Companies.

$$(v_A)_2 = \frac{h_{AB}}{r_A} = \frac{96.571\times10^9}{15\times10^6} = 6.4381\times10^3 \text{ m/s}$$

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{96.571\times10^9}{300\times10^6} = 321.90 \text{ m/s}$$

Second transfer orbit *BC*. $r_C = 9000 \text{ km} = 9\times10^6 \text{ m}$

At point *B*, where $\theta = 0$.

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At point *C*, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding, $$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_Br_C}{r_B + r_C}} = \sqrt{\frac{(2)\left(326.41\times10^{12}\right)\left(300\times10^6\right)\left(9\times10^6\right)}{309\times10^6}}$$

$$= 75.5265\times10^9 \text{ m}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{75.5265\times10^9}{300\times10^6} = 251.76 \text{ m/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{75.5265\times10^9}{9\times10^6} = 8.3918\times10^3 \text{ m/s}$$

Final circular orbit. $r_C = 9\times10^6 \text{ m}$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{326.41\times10^{12}}{9\times10^6}} = 6.0223\times10^3 \text{ m/s}$$


© 2007 The McGraw-Hill Companies.

Speed reductions.

(*a*) At *A*. $(v_A)_1 - (v_A)_2 = 6.5971 \times 10^3 - 6.4381 \times 10^3$

$$\Delta v_A = 159.0 \text{ m/s} \blacktriangleleft$$

(*b*) At *B*. $(v_B)_1 - (v_B)_2 = 321.90 - 251.76$

$$\Delta v_B = 70.1 \text{ m/s} \blacktriangleleft$$

(*c*) At *C*. $(v_C)_1 - (v_C)_2 = 8.3918 \times 10^3 - 6.0223 \times 10^3$

$$\Delta v_C = 2.37 \times 10^3 \text{ m/s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 109.

Data from Problem 12.108: $r_C = 9000$ km, $M = 0.82\ M_{\text{earth}}$

For Earth, $GM_{\text{earth}} = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.059 \times 10^{12}\ \text{m}^3/\text{s}^2$

For Venus, $GM = 0.82\,GM_{\text{earth}} = 326.409 \times 10^{12}\ \text{m}^3/\text{s}^2$

Transfer orbit $AB$: $v_A = 6500$ m/s, $r_A = 15 \times 10^6$ m

$$h_{AB} = r_A v_A = \left(15 \times 10^6\right)(6500) = 97.5 \times 10^9\ \text{m}^2/\text{s}$$

At point $A$, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C\cos 180° = \frac{GM}{h_{AB}^2} - C$$

At point $B$, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C\cos 0 = \frac{GM}{h_{AB}^2} + C$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h_{AB}^2}$$

$$\frac{1}{r_B} = \frac{2GM}{h_{AB}^2} - \frac{1}{r_A} = \frac{(2)\left(326.409 \times 10^{12}\right)}{\left(97.5 \times 10^9\right)^2} - \frac{1}{15 \times 10^6} = 2.0057 \times 10^{-9}\ \text{m}^{-1}$$

(*a*) Radial coordinate $r_B$. $r_B = 498.56 \times 10^6$ m $r_B = 499 \times 10^3$ km ◀

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{97.5 \times 10^9}{498.56 \times 10^6} = 195.56\ \text{m/s}$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

Second transfer orbit $BC$. $r_C = 9 \times 10^6$ m

At point $B$, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At point $C$, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding, $$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_B r_C}{r_B + r_C}} = \sqrt{\frac{(2)\left(326.409 \times 10^{12}\right)\left(498.56 \times 10^6\right)\left(9 \times 10^6\right)}{507.56 \times 10^6}}$$

$$= 75.968 \times 10^9 \text{ m}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{75.968 \times 10^9}{498.56 \times 10^6} = 152.37 \text{ m/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{75.968 \times 10^9}{9 \times 10^6} = 8.4409 \times 10^3 \text{ m/s}$$

Circular orbit with $r_C = 9 \times 10^6$ m.

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{326.409 \times 10^{12}}{9 \times 10^6}} = 6.0223 \times 10^3 \text{ m/s}$$

(*b*) Speed reductions at $B$ and $C$.

At $B$ $\quad (v_B)_1 - (v_B)_2 = 195.56 - 152.37$

$$\Delta v_B = 43.2 \text{ m/s} \blacktriangleleft$$

At $C$ $\quad (v_C)_1 - (v_C)_2 = 8.4409 \times 10^3 - 6.0223 \times 10^3$

$$\Delta v_C = 2.42 \times 10^3 \text{ m/s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 110.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For Mars, $GM = (0.1074)\left(14.077 \times 10^{15}\right) = 1.51188 \times 10^{15} \text{ ft}^3/\text{s}^2$

$$r_A = 5625 \text{ mi} = 29.7 \times 10^6 \text{ ft}, \qquad r_B = 112500 \text{ mi} = 594 \times 10^6 \text{ ft}$$

For the parabolic approach trajectory at $A$,

$$(v_A)_1 = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)\left(1.51188 \times 10^{15}\right)}{29.7 \times 10^6}} = 10.0901 \times 10^3 \text{ ft/s}$$

First elliptic transfer orbit $AB$.

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_A = -\cos\theta_B$.

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h_{AB}^2}$

$$h_{AB} = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(1.51188 \times 10^{15}\right)\left(29.7 \times 10^6\right)\left(594 \times 10^6\right)}{623.7 \times 10^6}}$$

$$h_{AB} = 292.45 \times 10^9 \text{ m}^2/\text{s}$$

$$a = \frac{1}{2}\left(r_A + r_B\right) = \frac{1}{2}\left(29.7 \times 10^6 + 594 \times 10^6\right) = 311.85 \times 10^6 \text{ ft}$$

$$b = \sqrt{r_Ar_B} = \sqrt{\left(29.7 \times 10^6\right)\left(594 \times 10^6\right)} = 132.82 \times 10^6 \text{ ft}$$

Periodic time for full ellipse: $\tau = \dfrac{2\pi ab}{h}$

For half ellipse $AB$, $\tau_{AB} = \dfrac{1}{2}\tau = \dfrac{\pi ab}{h}$

$$\tau_{AB} = \frac{\pi\left(311.85 \times 10^6\right)\left(132.82 \times 10^6\right)}{292.45 \times 10^9} = 444.95 \times 10^3 \text{ s} \qquad \tau_{AB} = 123.6 \text{ h} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 111.

From Keplers third law,

$$\left(\frac{T_H}{T_E}\right)^2 = \left(\frac{a_H}{a_E}\right)^3$$

Solve for $a_H$.

$$a_H = a_E\left(\frac{T_H}{T_E}\right)^{2/3} = r_E\left(\frac{76 \text{ years}}{1 \text{ year}}\right)^{2/3} = 17.942\ r_E$$

Also, $$a_H = \frac{1}{2}\left(r_{\min} + r_{\max}\right)$$

Solve for $r_{\max}$.

$$r_{\max} = 2a_H - r_{\min} = (2)(17.942\ r_E) - \frac{1}{2}r_E$$

$$= 35.3844\ r_E$$

or

$$r_{\max} = (35.3844)\left(92.6 \times 10^6 \text{ mi}\right)$$

$$r_{\max} = 3.28 \times 10^9 \text{ mi} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 112.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}$$

For circular orbit of satellite,

$$r_0 = 3960 + 310 = 4270 \text{ mi} = 22.546 \times 10^6 \text{ ft/s}$$

$$v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{14.077 \times 10^{15}}{22.546 \times 10^6}} = 24.988 \times 10^3 \text{ ft/s}$$

$$\tau_0 = \frac{2\pi r_0}{v_0} = \frac{(2\pi)\left(22.546 \times 10^6\right)}{24.988 \times 10^3} = 5.6692 \times 10^3 \text{ s}$$

For elliptic orbit of spacecraft it is given that

$$\tau = \frac{3}{2}\tau_0 = 8.5038 \times 10^3 \text{ s}$$

$$a = \frac{1}{2}(r_A + r_B), \qquad b = \sqrt{r_A r_B}$$

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_A = -\cos\theta_B.$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2a}{b^2} = \dfrac{2GM}{h^2}$ or $h = \sqrt{\dfrac{GMb^2}{a}}$

Periodic time: $\tau = \dfrac{2\pi ab}{h} = \dfrac{2\pi ab\sqrt{a}}{\sqrt{GMb^2}} = \dfrac{2\pi a^{3/2}}{\sqrt{GM}}$

$$a^3 = \frac{GM\tau^2}{4\pi^2} = \frac{\left(14.077 \times 10^{15}\right)\left(8.5038 \times 10^3\right)^2}{4\pi^2} = 25.786 \times 10^{21} \text{ ft}^3$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

$$a = 29.543 \times 10^6 \text{ ft}, \qquad r_A = r_0 = 22.546 \times 10^6 \text{ ft}$$

$$r_B = 2a - r_A = 36.541 \times 10^6 \text{ ft}, \qquad b = \sqrt{r_A r_B} = 28.703 \times 10^6 \text{ ft}$$

$$h = \frac{2\pi ab}{\tau} = \frac{2\pi\left(29.543 \times 10^6\right)\left(28.703 \times 10^6\right)}{8.5038 \times 10^3} = 626.54 \times 10^9 \text{ ft}^2/\text{s}$$

$$v_A = \frac{h}{r_A} = \frac{626.54 \times 10^9}{22.546 \times 10^6} = 27.789 \times 10^3 \text{ ft/s}$$

(*a*) Increase in speed at *A*.

$$\Delta v_A = v_A - v_0 = 27.789 \times 10^3 - 24.988 \times 10^3 \qquad \Delta v_A = 2.80 \times 10^3 \text{ ft/s} \blacktriangleleft$$

(*b*) Periodic time for elliptic orbit.

As calculated above $\tau = 8.5038 \times 10^3$ s $\qquad \tau = 141.7 \text{ min} \blacktriangleleft$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 113.

For earth's orbit about the sun,

$$v_0 = \sqrt{\frac{GM}{R_E}}, \quad \tau_0 = \frac{2\pi R_E}{v_0} = \frac{2\pi {R_E}^{3/2}}{\sqrt{GM}} \qquad \text{or} \qquad \sqrt{GM} = \frac{2\pi {R_E}^{3/2}}{\tau_0} \tag{1}$$

For the comet Hyakutake,

$$\frac{1}{r_0} = \frac{GM}{h^2} = (1+\varepsilon), \qquad \frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon), \qquad r_1 = \frac{1+\varepsilon}{1-\varepsilon} r_0$$

$$a = \frac{1}{2}(r_0 + r_1) = \frac{r_0}{1-\varepsilon}, \qquad b = \sqrt{r_0 r_1} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\, r_0$$

$$h = \sqrt{GMr_0(1+\varepsilon)}$$

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi {r_0}^2 (1+\varepsilon)^{1/2}}{(1-\varepsilon)^{3/2}\sqrt{GMr_0(1+\varepsilon)}}$$

$$= \frac{2\pi {r_0}^{3/2}}{\sqrt{GM(1-\varepsilon)^{3/2}}} = \frac{2\pi {r_0}^{3/2}\tau_0}{2\pi {R_E}^3 (1-\varepsilon)^{3/2}}$$

$$= \left(\frac{r_0}{R_E}\right)^{3/2} \frac{1}{(1-\varepsilon)^{3/2}}\ \tau_0$$

$$= (0.230)^{3/2} \frac{1}{(1-0.999887)^{3/2}}\ \tau_0 = 91.8 \times 10^3\ \tau_0$$

Since $\tau_0 = 1$ yr, $\tau = (91.8 \times 10^3)(1.000)$ $\qquad \tau = 91.8 \times 10^3$ yr ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 114.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

At point $A$, $r_A = 4560 \text{ mi} = 24.077 \times 10^6 \text{ ft}$

$$v_A = 6.48 \text{ mi/s} = 34.214 \times 10^3 \text{ ft/s}$$

from which $h = r_A v_A = 823.78 \times 10^9 \text{ ft}^2/\text{s}$

For trajectory $BAC$, $\dfrac{1}{r} = \dfrac{GM}{h^2}\left(1 + \varepsilon \cos\theta\right)$ with $\varepsilon = 1$

At point $A$, $\theta = 0$ while at $B$ and $C$, $\theta = \pm 90°$

$$\frac{1}{r_B} = \frac{1}{r_C} = \frac{GM}{h^2} \qquad \text{or} \qquad r_B = r_C = \frac{h^2}{GM} = 2r_A$$

As the spacecraft travels from $B$ to $C$, the area swept out is a parabolic area $A$.

$$A = \frac{2}{3}\left(r_B + r_C\right) r_A = \frac{8}{3} r_A{}^2 = \frac{8}{3}\left(24.077 \times 10^6\right)^2 = 15.458 \times 10^{15} \text{ ft}^2$$

$$\frac{dA}{dt} = \frac{1}{2}h \qquad \text{or} \qquad A = \frac{1}{2}\int h \, dt = \frac{1}{2}ht$$

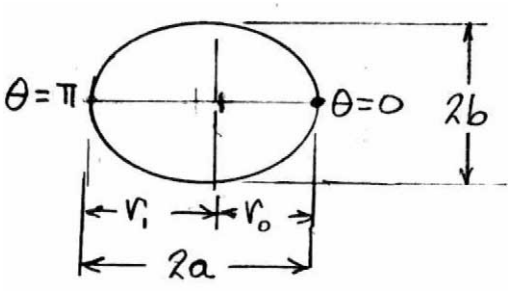
$$t_{BC} = \frac{2A}{h} = \frac{(2)\left(15.458 \times 10^{15}\right)}{823.78 \times 10^9} = 3.753 \times 10^3 \text{ s} \qquad \tau_{BC} = 1.043 \text{ h} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 115.

For earth, $g = 32.2 \text{ ft/s}^2$, $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)\left(20.9088 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$




For the orbit, $r_0 = 3960 + 182 = 4142 \text{ mi} = 21.8698 \times 10^6 \text{ ft}$

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \qquad \frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon)$$

$$r_1 = r_0\frac{1+\varepsilon}{1-\varepsilon} - \left(21.8698 \times 10^6\right)\frac{1.0356}{0.9644} = 23.4844 \times 10^6 \text{ ft}$$

$$a = \frac{1}{2}(r_0 + r_1) = 22.6770 \times 10^6 \text{ ft}$$

$$b = \sqrt{r_0 r_1} = 22.6627 \times 10^6 \text{ ft}$$

$$h = \sqrt{(1+\varepsilon)GMr_0} = \sqrt{(1.0356)\left(14.077 \times 10^{15}\right)\left(21.8698 \times 10^6\right)}$$

$$= 564.64 \times 10^9 \text{ ft}^2/\text{s}$$

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi\left(22.6770 \times 10^6\right)\left(22.6627 \times 10^6\right)}{564.64 \times 10^9}$$

$$= 5.7188 \times 10^3 \text{ s} \qquad \tau = 95.3 \text{ min} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 116.

For the circular orbit, $r_0 = r_A = nR$

$$v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{GM}{nR}}$$

The crash trajectory is elliptic.

$$v_A = \beta v_0 = \sqrt{\frac{\beta^2 GM}{nR}}$$

$$h = r_A v_A = nRv_A = \sqrt{\beta^2 n\, GMR}$$

$$\frac{GM}{h^2} = \frac{1}{\beta^2 nR}$$

$$\frac{1}{r} = \frac{GM}{h^2}\left(1 + \varepsilon\cos\theta\right) = \frac{1 + \varepsilon\cos\theta}{\beta^2 nR}$$

At point $A$, $\theta = 180°$

$$\frac{1}{r_A} = \frac{1}{nR} = \frac{1 - \varepsilon}{\beta^2 nR} \qquad \text{or} \qquad \beta^2 = 1 - \varepsilon \qquad \text{or} \qquad \varepsilon = 1 - \beta^2$$

At impact point $B$, $\theta = \pi - \phi$

$$\frac{1}{r_B} = \frac{1}{R}$$

$$\frac{1}{R} = \frac{1 + \varepsilon\cos\left(\pi - \phi\right)}{\beta^2 nR} = \frac{1 - \varepsilon\cos\phi}{\beta^2 nR}$$

$$\varepsilon\cos\phi = 1 - n\beta^2 \qquad \text{or} \qquad \cos\phi = \frac{1 - n\beta^2}{\varepsilon} = \frac{1 - n\beta^2}{1 - \beta^2}$$

$$\phi = \cos^{-1}\left[\left(1 - n\beta^2\right)/\left(1 - \beta^2\right)\right] \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 117

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For the moon, $GM = (0.01230)\left(14.077 \times 10^{15}\right) = 173.149 \times 10^{12} \text{ ft}^3/\text{s}^2$

For elliptic orbit $AB$, $r_A = 1110 \text{ mi} = 5.861 \times 10^6 \text{ ft}$, $r_B = 2240 \text{ mi} = 11.827 \times 10^6 \text{ ft}$

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_B = -\cos\theta_A$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(173.149 \times 10^{12}\right)\left(5.861 \times 10^6\right)\left(11.827 \times 10^6\right)}{17.688 \times 10^6}} = 36.839 \times 10^9 \text{ ft}^2/\text{s}$$

$$\left(v_B\right)_1 = \frac{h_{AB}}{r_A} = \frac{36.839 \times 10^9}{11.827 \times 10^6} = 3.1148 \times 10^3 \text{ ft/s}$$

For crash trajectory $BC$, $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon\cos\theta)$

At $B$, $\theta = 180°$, $r = r_B$, $\dfrac{1}{r_B} = \dfrac{GM}{h_{BC}^2}(1 - \varepsilon)$ (1)

At $C$, $\theta = 70°$, $r = r_C$, $\dfrac{1}{r_C} = \dfrac{GM}{h_{BC}^2}(1 + \varepsilon\cos 70°)$ (2)


© 2007 The McGraw-Hill Companies.

Dividing Eq. (2) by Eq. (1),

$$\frac{r_B}{r_C} = \frac{1 + \varepsilon \cos 70°}{1 - \varepsilon} \qquad \text{or} \qquad \varepsilon = \frac{r_B / r_C - 1}{(r_B / r_C) + \cos 70°}$$

$$\varepsilon = \frac{2240/1080 - 1}{2240/1080 + \cos 70°} = 0.44455$$

From Eq. (1), $h_{BC} = \sqrt{GM(1 - \varepsilon) r_B} = \sqrt{\left(173.149 \times 10^{12}\right)(0.55545)\left(11.827 \times 10^{6}\right)} = 33.726 \times 10^{9}$ ft$^2$/s

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{33.726 \times 10^{9}}{11.827 \times 10^{6}} = 2.8516 \times 10^{3} \text{ ft/s}$$

$$\Delta v_B = (v_B)_2 - (v_B)_1 = -263.2 \text{ ft/s} \qquad |\Delta v_B| = 263 \text{ ft/s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 118.

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)\left(20.909 \times 10^6\right)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For the trajectory, $r_C = 3960 + 930 = 4890 \text{ mi} = 25.819 \times 10^6 \text{ ft}$

$$r_A = r_B = R = 20.909 \times 10^6 \text{ ft}, \qquad \frac{r_C}{r_A} = \frac{25.819 \times 10^6}{20.909 \times 10^6} = 1.2348$$

Range $A$ to $B$: $s_{AB} = 3700 \text{ mi} = 19.536 \times 10^6 \text{ ft}$

$$\phi = \frac{s_{AB}}{R} = \frac{19.536 \times 10^6}{20.909 \times 10^6} = 0.9343 \text{ rad} = 53.534°$$

For an elliptic trajectory, $\dfrac{1}{r} = \dfrac{GM}{h^2}\left(1 + \varepsilon\cos\theta\right)$

At $A$, $\theta = 180° - \dfrac{\phi}{2} = 153.233°$, $\dfrac{1}{r_A} = \dfrac{GM}{h^2}\left(1 + \varepsilon\cos 153.233°\right)$ (1)

At $C$, $\theta = 180°$, $\dfrac{1}{r_C} = \dfrac{GM}{h^2}\left(1 - \varepsilon\right)$ (2)

Dividing Eq. (1) by Eq. (2),

$$\frac{r_C}{r_A} = \frac{1 + \varepsilon\cos 153.233°}{1 - \varepsilon} = 1.2348$$

$$\varepsilon = \frac{1.2348 - 1}{1.2348 + \cos 153.233°} = 0.6866$$

From Eq. (2), $h = \sqrt{GM\left(1 - \varepsilon\right)r_C}$

$$h = \sqrt{\left(14.077 \times 10^{15}\right)\left(0.3134\right)\left(25.819 \times 10^6\right)} = 337.500 \times 10^9 \text{ ft}^2/\text{s}$$

(*a*) Velocity at $C$. $v_C = \dfrac{h}{r_C} = \dfrac{337.5 \times 10^9}{25.819 \times 10^6} = 13.07 \times 10^3 \text{ ft/s}$ $v_C = 2.48 \text{ mi/s}$ ◀

(*b*) Eccentricity of trajectory. $\varepsilon = 0.687$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 119.

Radius of Earth $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

At point $A$, $r_A = R + \text{altitude} = 4160 \text{ mi} = 21.9648 \times 10^6 \text{ ft}$

For Earth, $GM = gR^2 = (32.2)\left(20.9088 \times 10^6\right)^2 = 14.0771 \times 10^{15} \text{ ft}^3/\text{s}^2$

For circular orbit at point $A$,

$$\left(v_A\right)_1 = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.0771 \times 10^{15}}{21.9648 \times 10^6}} = 25.3159 \times 10^3 \text{ ft/s}$$

After speed reduction at $A$,

$$\left(v_A\right)_2 = (0.94)\left(25.3159 \times 10^3\right) = 23.7969 \times 10^3 \text{ ft/s}$$

For trajectory $AB$,

$$h_{AB} = r_A\left(v_A\right)_2 = \left(21.9648 \times 10^6\right)\left(23.7969 \times 10^3\right)$$

$$= 522.6946 \times 10^9 \text{ ft}^2/\text{s}$$

At point $A, \theta = 0$.

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C \cos\theta = \frac{GM}{h_{AB}^2} + C$$

Solving for $C$,

$$C = \frac{1}{r_A} - \frac{GM}{h_{AB}^2} = \frac{1}{21.9648 \times 10^6} - \frac{14.0771 \times 10^{15}}{\left(522.6946 \times 10^9\right)^2}$$

$$= -5.997512 \times 10^{-9} \text{ ft}^{-1}$$

*continued*

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

At point $B, \theta = 50°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C \cos 50°$$

$$= \frac{14.0771 \times 10^{15}}{\left(522.6946 \times 10^{9}\right)^2} + \left(-5.997512 \cos 50°\right)$$

$$= 47.6698 \times 10^{-9} \text{ ft}^{-1}$$

$$r_B = 20.9777 \times 10^{6} \text{ ft}$$

Altitude at $B$. $r_B - R = 68.9 \times 10^{3}$ ft altitude = 13.05 mi ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 120.

Using Eq. (12.39), $\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$ and $\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B.$

Adding, $\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$

At points $A$ and $B$ the radial direction is normal to the path.

$$a_n = \frac{v^2}{\rho} = \frac{h^2}{r^2\rho}$$

But $F_n = \frac{GMm}{r^2} = ma_n = \frac{mh^2}{r^2\rho}$

$$\frac{1}{\rho} = \frac{GM}{h^2} = \frac{1}{2}\left(\frac{1}{r_A} + \frac{1}{r_B}\right) \qquad \frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right) \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 121.

$$\frac{1}{r} = \frac{GM}{h^2}\left(1 + \varepsilon\cos\theta\right)$$

At $A$, $\theta = 0$ $\dfrac{1}{r_A} = \dfrac{GM}{h^2} = \left(1 + \varepsilon\right)$ or $r_A = \dfrac{h^2}{GM\left(1 + \varepsilon\right)}$

At $B$, $\theta = 180°$ $\dfrac{1}{r_B} = \dfrac{GM}{h^2} = \left(1 - \varepsilon\right)$ or $r_B = \dfrac{h^2}{GM\left(1 - \varepsilon\right)}$

$$\frac{r_B}{r_A} = \frac{1 + \varepsilon}{1 - \varepsilon} = \frac{r_1}{r_0}$$

($a$) $$\varepsilon = \frac{r_1 - r_0}{r_1 + r_0}$$ ◀

$$R_E = 93 \times 10^6 \text{ mi}$$

$$r_0 = 0.230\, R_E = 21.39 \times 10^6 \text{ mi}$$

($b$) $$r_1 = \frac{1 + \varepsilon}{1 - \varepsilon} r_0 = \frac{1 + 0.999887}{1 - 0.999887}\left(21.39 \times 10^6\right)$$ $$r_1 = 379 \times 10^9 \text{ mi}$$ ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 122.

For an ellipse $2a = r_A + r_B$ and $b = \sqrt{r_A r_B}$

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B.$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2a}{b^2} = \dfrac{2GM}{h^2}$

$$h = b\sqrt{\frac{GM}{a}}$$

By Eq. (12.45),

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi ab\sqrt{a}}{b\sqrt{GM}} = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\tau^2 = \frac{4\pi^2 a^3}{GM}$$

For orbits 1 and 2 about the same large mass,

$$\tau_1^2 = \frac{4\pi^2 a_1^3}{GM} \qquad \text{and} \qquad \tau_2^2 = \frac{4\pi^2 a_2^3}{GM}$$

Forming the ratio, $\left(\dfrac{\tau_1}{\tau_2}\right)^2 = \left(\dfrac{a_1}{a_2}\right)^3$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 123.

By Eq. $(12.39')$, $\dfrac{1}{r} = \dfrac{GM}{h^2}\left(1 + \varepsilon\cos\theta\right)$

At $A$, $\theta = 0°$ $\dfrac{1}{r_A} = \dfrac{GM}{h^2} = \left(1 + \varepsilon\right)$ or $r_A = \dfrac{h^2}{GM\left(1 + \varepsilon\right)}$

At $B$, $\theta = 180°$ $\dfrac{1}{r_B} = \dfrac{GM}{h^2} = \left(1 - \varepsilon\right)$ or $r_B = \dfrac{h^2}{GM\left(1 - \varepsilon\right)}$

Adding, $r_A + r_B = \dfrac{h^2}{GM} = \left(\dfrac{1}{1+\varepsilon} + \dfrac{1}{1-\varepsilon}\right) = \dfrac{2h^2}{GM\left(1-\varepsilon^2\right)}$

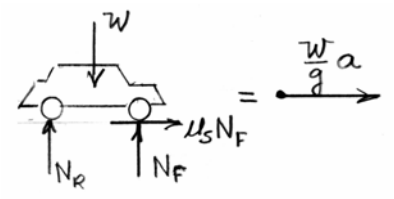
But for an ellipse, $r_A + r_B = 2a$

$$2a = \frac{2h^2}{GM\left(1-\varepsilon^2\right)} \qquad h = \sqrt{GMa\left(1-\varepsilon^2\right)} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 124.

(*a*) Front wheel drive with $N_f = 0.65W$.




$$\pm \Sigma F = ma: \quad \mu_s N_F = \frac{W}{g}a$$

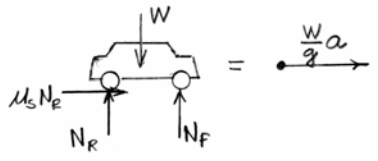
$$a = \frac{g\mu_s N_F}{W} = \frac{(32.2)(0.80)(0.65W)}{W} = 16.74 \text{ ft/s}^2$$

For constant acceleration,

$$v_{\max}^2 = 2ax = (2)(16.74)(1320) = 44.204 \times 10^3 \text{ ft}^2/\text{s}^2$$

$$v_{\max} = 210 \text{ ft/s} \qquad v_{\max} = 143.2 \text{ mi/h} \blacktriangleleft$$

(*b*) Rear wheel drive with $N_R = 0.42\,\text{W}$.




$$\pm \Sigma F = ma: \quad \mu_s N_R = \frac{W}{g}a$$

$$a = \frac{g\mu_s N_R}{W} = \frac{(32.2)(0.80)(0.42W)}{W} = 10.82 \text{ ft/s}^2$$

For constant acceleration,

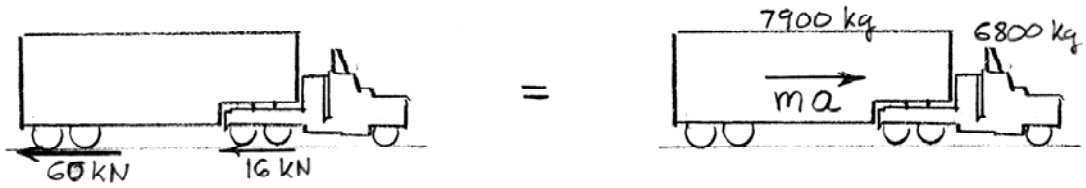
$$v_{\max}^2 = 2ax = (2)(10.82)(1320) = 28.563 \times 10^3 \text{ ft}^2/\text{s}^2$$

$$v_{\max} = 169.0 \text{ ft/s} \qquad v_{\max} = 115.2 \text{ mi/h} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 125.

$$v_0 = 90 \text{ km/h} = 25 \text{ m/s}$$

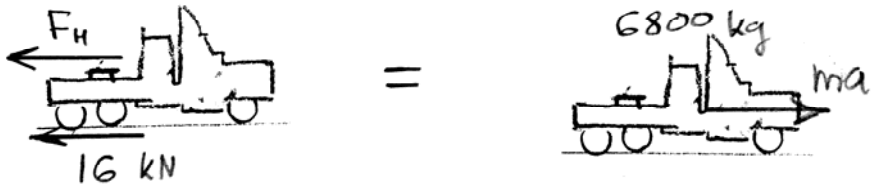



$$\pm\Sigma F_x = ma: \quad -60\times10^3 - 16\times10^3 = (7900 + 6800)a$$

$$a = -\frac{76\times10^3}{14700} = -5.170 \text{ m/s}^2$$

For constant acceleration, $\dfrac{v^2}{2} - \dfrac{v_0^2}{2} = a(x - x_0)$

(*a*) $x - x_0 = \dfrac{v^2 - v_0^2}{2a} = \dfrac{0 - (25)^2}{(2)(-5.170)}$ $\qquad x - x_0 = 60.4 \text{ m}$ ◀




$$\pm\Sigma F_x = ma: \quad -F_H - 16\times10^3 = (6800)(-5.170)$$

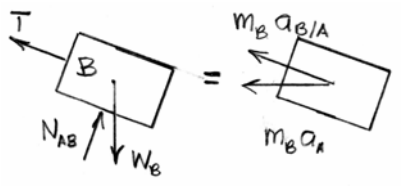
(*b*) $F_H = (6800)(5.170) - 16\times10^3 = 19.16\times10^3 \text{ N}$ $\qquad F_H = 19.16 \text{ kN}$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 126.

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the inclined contact surface.

Block $B$: $+\nwarrow \Sigma F_x = \Sigma m a_x$:




$$T - W_B \sin 25° = m_B a_A \cos 25° + m_B a_{B/A}$$

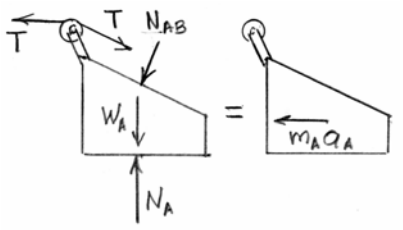
$$\left(\frac{30}{32.2}\cos 25°\right) a_A + \frac{30}{32.2} a_{B/A} = 50 - 30 \sin 25°$$

$$0.84439\, a_A + 0.93168\, a_{B/A} = 37.321 \qquad (1)$$

$$+\nearrow \Sigma F_y = \Sigma m_B a_y: \quad N_{AB} - W_B \cos 25° = -m_B a_A \sin 25°$$

$$\left(\frac{30}{32.2}\sin 25°\right) a_A + N_{AB} = 30 \cos 25° \text{ or } 0.39374\, a_A + N_{AB} = 27.189 \qquad (2)$$

Block $A$: $\overset{+}{\leftarrow} \Sigma F_x = m a_x$: $T - T\cos 25° + N_{AB} \sin 25° = m_A a_A$




$$\frac{55}{32.2} a_A - (\sin 25°) N_{AB} = 50(1 - \cos 25°)$$

$$1.70807\, a_A - 0.42262 N_{AB} = 4.6846 \qquad (3)$$

Using (2) and (3) to eliminate $N_{AB}$ and solve for $a_A$,

(*a*) Acceleration of block $A$. $\mathbf{a}_A = 8.63 \text{ ft/s}^2 \leftarrow$ ◀

Substituting for $a_A$ into (1) and solving for $a_{B/A}$,

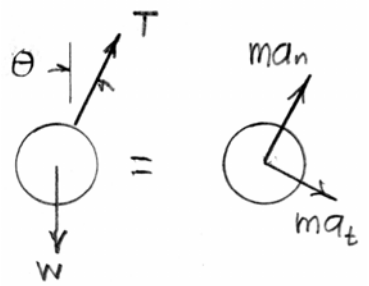
(*b*) Acceleration of $B$ relative to $A$. $\mathbf{a}_{B/A} = 32.2 \text{ ft/s}^2 \searrow 25°$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 127.

$$m = \frac{W}{g} = \frac{180}{32.2} = 5.59 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\rho = 40 \text{ ft}$$




$$+\nearrow \Sigma F_n = ma_n: \quad T - W\cos\theta = ma_n = \frac{mv^2}{\rho}$$

$$T = \frac{mv^2}{\rho} + W\cos\theta$$

(*a*) At top of swing. $\theta = 30°$, $v = 0$

$$T = 0 + 180\cos 30° = 155.9 \qquad T = 155.9 \text{ lb} \blacktriangleleft$$

(*b*) At bottom of swing. $\theta = 0°$, $v = 18.6$ ft/s

$$T = \frac{(5.59)(18.6)^2}{40} + 180 = 228 \qquad T = 228 \text{ lb} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

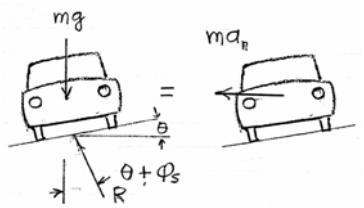
**Chapter 12, Solution 128.**

The road reaction consists of normal component **N** and friction component **F**. The resultant **R** makes angle $\phi_s$ with the normal.

$$\tan \phi_s = \mu_s = 0.65 \qquad \text{or} \qquad \phi_s = 33.02°$$

$$v_0 = 110 \text{ km/h} = 30.56 \text{ m/s}$$

($a$) $\theta = 10°$.




$$\Sigma F_y = 0: \quad R\cos(\theta + \phi_s) - mg = 0$$

$$R = \frac{mg}{\cos(\theta + \phi_s)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_n: \quad R\sin(\theta + \phi_s) = ma_n$$

$$ma_n = mg\tan(\theta + \phi_s)$$

$$\frac{v_{\max}^2}{r} = g\tan(\theta + \phi_s)$$

$$v_{\max} = \sqrt{rg\tan(\theta + \phi_s)} = \sqrt{(50)(9.81)\tan(10° + 33.02°)} = 21.39 \text{ m/s}$$

Speed reduction: $\Delta v = v_0 - v_{\max} = 30.56 - 21.39 = 9.16$ m/s

$$\Delta v = 33.0 \text{ km/h} \blacktriangleleft$$

($b$) $\theta = -5°$.

The equation derived in part ($a$) applies also for negative value of $\theta$.

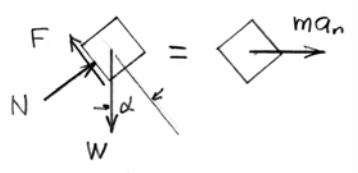
$$v_{\max} = \sqrt{rg\tan(\theta + \phi_s)} = \sqrt{(50)(9.81)\tan(-5° + 33.02°)} = 16.16 \text{ m/s}$$

Speed reduction: $\Delta v = v_0 - v_{\max} = 30.56 - 16.16 = 14.40$ m/s

$$\Delta v = 51.8 \text{ km/h} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 129.




$$\alpha = 40°, \quad r = 24 \text{ in.} = 2 \text{ ft}, \quad \mu_s = 0.35$$

$F = \pm\mu_s N$, where the upper sign applies for downward impending motion and the lower sign for upward impending motion.

$$+\Sigma F_y = 0: \quad N\sin\alpha + F\cos\alpha - W = 0$$

$$N\left(\sin\alpha \pm \mu_S \cos\alpha\right) = W$$

$$N = \frac{W}{\sin\alpha \pm \mu_S \cos\alpha}$$

$$\xrightarrow{+} \Sigma F_x = ma_n: \quad N\cos\alpha - F\sin\alpha = \frac{W}{g}\frac{v^2}{r}$$

$$N\left(\cos\alpha \mp \mu_s \sin\alpha\right) = \frac{W}{g}\frac{v^2}{r}$$

$$v^2 = gr\frac{N\left(\cos\alpha \mp \mu_s \sin\alpha\right)}{W} = gr\frac{\cos\alpha \mp \mu_s \sin\alpha}{\sin\alpha \pm \mu_s \cos\alpha}$$

For impending motion downward,

$$v^2 = (32.2)(2.0)\frac{\cos 40° - 0.35\sin 40°}{\sin 40° + 0.35\cos 40°} = 38.25 \text{ ft}^2/\text{s}^2$$

$$v = 6.18 \text{ ft/s}$$

For impending motion upward,

$$v^2 = (32.2)(2.0)\frac{\cos 40° + 0.35\sin 40°}{\sin 40° - 0.35\cos 40°} = 170.34 \text{ ft}^2/\text{s}^2$$

$$v = 13.05 \text{ ft/s}$$

Range of speeds for which the collar will not slide:

$$6.18 \text{ ft/s} \le v \le 13.05 \text{ ft/s} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 130.

$$r = \left(0.5 + 0.3\sin \pi t\right)\text{m} \qquad \theta = 2\pi\left(t^2 - 2t\right) \text{ rad}$$

$$\dot{r} = 0.3\pi \cos \pi t \text{ m/s} \qquad \dot{\theta} = 4\pi\left(t - 1\right) \text{ rad/s}$$

$$\ddot{r} = -0.3\pi^2 \sin \pi t \qquad \ddot{\theta} = 4\pi \text{ rad/s}^2$$

(*a*) At $t = 0$, $r = 0.5$ m $\qquad \theta = 0$

$$\dot{r} = 0.94248 \text{ m/s} \qquad \dot{\theta} = -12.5664 \text{ rad/s}$$

$$\ddot{r} = 0 \qquad \ddot{\theta} = 12.5664 \text{ rad/s}^2$$

$$a_r = \ddot{r} - \dot{r}\dot{\theta}^2 = 0 - \left(0.5\right)\left(-12.5664\right)^2 = -78.957 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(0.5\right)\left(12.5664\right) + \left(2\right)\left(0.94248\right)\left(-12.5664\right) = -17.4040 \text{ m/s}$$

$$F_r = m_B a_r: \quad F_r = \left(0.4\right)\left(-78.957\right) = -31.6 \qquad F_r = -31.6 \text{ N} \blacktriangleleft$$

$$F_\theta = m_B a_\theta: \quad F_\theta = \left(0.4\right)\left(-17.4040\right) = -6.96 \qquad F_\theta = -6.96 \text{ N} \blacktriangleleft$$

(*b*) At $t = 0.8$ s, $r = 0.67634$ m $\qquad \theta = -6.0319$ rad

$$\dot{r} = -0.76248 \text{ m/s} \qquad \dot{\theta} = -2.5133 \text{ rad/s}$$

$$\ddot{r} = -1.74040 \text{ m/s}^2 \qquad \ddot{\theta} = 12.5664 \text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -1.74040 - \left(0.67634\right)\left(-2.5133\right)^2 = -6.0126 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(0.67634\right)\left(12.5664\right) + \left(2\right)\left(-0.76248\right)\left(-2.5133\right) = 12.332 \text{ m/s}^2$$

$$F_r = m_B a_r: \quad F_r = \left(0.4\right)\left(-6.0126\right) = -2.41 \qquad F_r = -2.41 \text{ N} \blacktriangleleft$$

$$F_\theta = m_B a_\theta: \quad F_\theta = \left(0.4\right)\left(12.332\right) = 4.93 \qquad F_\theta = 4.93 \text{ N} \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 131.

$$\dot{\theta} = 10 \text{ rad/s}, \quad \ddot{\theta} = 0, \quad b = 9 \text{ in.} = 0.75 \text{ ft}, \quad W = 4 \text{ oz} = 0.25 \text{ lb}$$

Kinematics: $r = \dfrac{b}{\cos\theta}, \quad \dot{r} = \dfrac{b\sin\theta}{\cos^2\theta}\dot{\theta}$

$$\ddot{r} = \frac{b\sin\theta}{\cos^2\theta}\ddot{\theta} + \frac{b\left[\left(\cos^2\theta\right)\left(\cos\theta\right) - \left(\sin\theta\right)\left(2\cos\theta\right)\left(-\sin\theta\right)\right]}{\cos^4\theta}\dot{\theta}^2$$

$$= \frac{b\left(1+\sin^2\theta\right)}{\cos^3\theta} \quad \text{with} \quad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{b\left(1+\sin^2\theta\right)}{\cos^3\theta}\dot{\theta}^2 - \frac{b}{\cos\theta}\dot{\theta}^2 = \frac{2b\sin^2\theta}{\cos^3\theta}\dot{\theta}^2$$

$$= 2b\tan^2\theta\sec\theta\,\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\frac{b\sin\theta}{\cos^2\theta}\dot{\theta}^2 = 2b\tan\theta\sec\theta\,\dot{\theta}^2$$

(*a*) Radial and transverse components of effective forces.

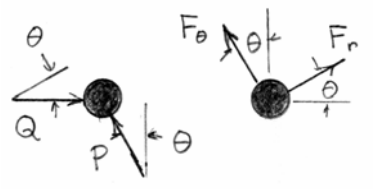
$$F_r = \frac{W}{g}a_r: \quad F_r = \frac{2Wb}{g}\tan^2\theta\sec\theta\,\dot{\theta}^2$$

$$= \frac{(2)(0.25)(10)^2}{32.2}\tan^2\theta\sec\theta \qquad F_r = 1.165\tan^2\theta\sec\theta \text{ lb} \blacktriangleleft$$

$$F_\theta = \frac{W}{g}a_\theta: \quad F_\theta = \frac{2Wb}{g}\tan\theta\sec\theta\,\dot{\theta}$$

$$= \frac{(2)(0.25)(10)^2}{32.2}\tan\theta\sec\theta \qquad F_\theta = 1.165\tan\theta\sec\theta \text{ lb} \blacktriangleleft$$

(*b*) Forces **P** and **Q** exerted on the pin by the arm *OA* and the wall of the slot *DE*, respectively.




$$\Sigma F_y = P\cos\theta = F_r\sin\theta - F_\theta\cos\theta$$

$$\mathbf{P} = 1.165\tan\theta\sec^3\theta \text{ lb} \measuredangle \theta \blacktriangleleft$$

$$+\nearrow \Sigma F_r = Q\cos\theta = F_r$$

$$\mathbf{Q} = 1.165\tan^2\theta\sec^2\theta \text{ lb} \rightarrow \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 132.

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

Solving for $M$, $\quad M = \dfrac{4\pi^2 r^3}{G\tau^2}$

Data: $r = 384.5 \times 10^6$ m

$\tau = 27.32$ days $= 655.68$ h $= 2.3604 \times 10^6$ s

$G = 66.73 \times 10^{-12}$ m$^3$/kg·s$^2$

$$M = \frac{4\pi^2\left(384.5 \times 10^6\right)^3}{\left(66.73 \times 10^{-12}\right)\left(2.3604 \times 10^6\right)^2} = 6.04 \times 10^{24}$$

$M = 6.04 \times 10^{24}$ kg ◀

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 133.

For earth: $R_e = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}, \quad g_e = 9.81 \text{ m/s}^2$

For the given orbit: $r_e = 6370 + 4500 = 10870 \text{ km} = 10.870 \times 10^6 \text{ m}$

$$(GM)_e = g_e R_e^{\,2} = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

(*a*) $$F = \frac{(GM)_e m}{r^2} = \frac{\left(398.06 \times 10^{12}\right)(540)}{\left(10.870 \times 10^6\right)^2} = 1819 \text{ N}$$

$F = 1.819 \text{ kN}$ ◀

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let $\tau$ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r \qquad \text{or} \qquad \tau^2 = \frac{4\pi^2 r^3}{GM}$$

Since earth orbit and moon orbit have the same periodic time,

$$\tau^2 = \frac{4\pi^2 r_e^{\,3}}{GM_e} = \frac{4\pi^2 r_m^{\,3}}{GM_m}$$

(*b*) $$r_m = \left(\frac{M_m}{M_e}\right)^{1/3} r_e = \left(0.01230\right)^{1/3}\left(10.870 \times 10^6\right) = 2.509 \times 10^6 \text{ m}$$

$r_B = 2510 \text{ km}$ ◀

$$GM_e = g_e R_e^{\,2} \qquad \text{and} \qquad GM_m = g_m R_m^{\,2}$$

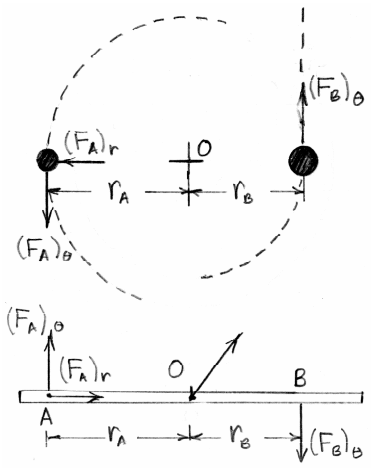
$$G = \frac{g_e R_e^{\,2}}{M_e} = \frac{g_m R_m^{\,2}}{M_m} \quad \text{with} \quad R_m = 1740 \text{ km} = 1.740 \times 10^6 \text{ m}$$

(*c*) $$g_m = \left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)^2 g_e = \left(0.01230\right)\left(\frac{6.37 \times 10^6}{1.740 \times 10^6}\right)^2 (9.81)$$

$g_m = 1.617 \text{ m/s}^2$ ◀


© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 134.




Let r and $\theta$ be polar coordinates with the origin lying at the shaft.

Constraint of rod: $\theta_B = \theta_A + \pi$ radians; $\dot{\theta}_B = \dot{\theta}_A = \dot{\theta}$; $\ddot{\theta}_B = \ddot{\theta}_A = \ddot{\theta}$.

(*a*) Components of acceleration

Sketch the free body diagrams of the balls showing the radial and transverse components of the forces acting on them. Owing to frictionless sliding of *B* along the rod, $(F_B)_r = 0$.

Radial component of acceleration of *B.*

$$F_r = m_B(a_B)_r: \qquad (a_B)_r = 0 \blacktriangleleft$$

Transverse components of acceleration.

$$(a_A)_\theta = r_A\ddot{\theta} + 2\dot{r}_A\dot{\theta} = ra\ddot{\theta}$$

$$(a_B)_\theta = r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta} \qquad (1)$$

Since the rod is massless, it must be in equilibrium. Draw its free body diagram, applying Newton's 3rd Law.

$$+\circlearrowleft \Sigma M_0 = 0: \quad r_A(F_A)_\theta + r_B(F_B)_\theta = r_A m_A(a_A)_\theta + r_B m_B(a_B)_\theta = 0$$

$$r_A m_A r_A\ddot{\theta} + r_B m_B\left(r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta}\right) = 0$$

$$\ddot{\theta} = \frac{-2r_B\dot{r}_B\dot{\theta}}{m_A r_A^{\,2} + m_B r_B^{\,2}}$$

At $t = 0$, $\dot{r}_B = 0$ so that $\ddot{\theta} = 0$.

From Eq. (1), $(a_B)_\theta = 0 \blacktriangleleft$

(*b*) Acceleration of *B* relative to the rod.

At $t = 0$, $(v_A)_\theta = 8$ ft/s $= 96$ in./s, $\dot{\theta} = \dfrac{(v_A)_\theta}{r_A} = \dfrac{96}{10} = 9.6$ rad/s

$$\ddot{r}_B - r_B\dot{\theta}^2 = (a_B)_r = 0$$

$$\ddot{r}_B = r_B\dot{\theta}^2 = (8)(9.6)^2 = 737.28 \text{ in./s}^2$$

$$\ddot{r}_B = 61.4 \text{ ft/s}^2 \blacktriangleleft$$

*Continued*


© 2007 The McGraw-Hill Companies.

(*c*) Speed of *A*.

Substituting $\frac{d}{dt}\left(mr^2\dot{\theta}\right)$ for $rF_\theta$ in each term of the moment equation gives

$$\frac{d}{dt}\left(m_A r_A^{\,2}\dot{\theta}\right) + \frac{d}{dt}\left(m_B r_B^{\,2}\dot{\theta}\right) = 0$$

Integrating with respect to time,

$$m_A r_A^{\,2}\dot{\theta} + m_B r_B^{\,2}\dot{\theta} = \left(m_A r_A^{\,2}\dot{\theta}\right)_0 + \left(m_B r_B^{\,2}\dot{\theta}\right)_0$$

Applying to the final state with ball *B* moved to the stop at *C*,

$$\left(\frac{W_A}{g}r_A^2 + \frac{W_B}{g}r_C^2\right)\dot{\theta}_f = \left[\frac{W_A}{g}r_A^2 + \frac{W_B}{g}\left(r_B\right)_0^2\right]\dot{\theta}_0$$

$$\dot{\theta}_f = \frac{W_A r_A^{\,2} + W_B\left(r_B\right)_0^2}{W_A r_A^{\,2} + W_B r_C^{\,2}}\dot{\theta}_0 = \frac{(1)(10)^2 + (2)(8)^2}{(1)(10)^2 + (2)(16)^2}(9.6) = 3.5765 \text{ rad/s}$$

$$\left(v_A\right)_f = r_A\dot{\theta}_f = (10)(3.5765) = 35.765 \text{ in./s} \qquad \left(v_A\right)_f = 2.98 \text{ ft/s} \blacktriangleleft$$

*Vector Mechanics for Engineers: Statics and Dynamics*, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
© 2007 The McGraw-Hill Companies.

## Chapter 12, Solution 135.

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$r_A = 6370 + 563 = 6933 \text{ km} = 6.933 \times 10^6 \text{ m}$$

$$r_B = 6370 + 121 = 6491 \text{ km} = 6.491 \times 10^6 \text{ m}$$

For the circular orbit through point $A$,

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.06 \times 10^{12}}{6.933 \times 10^6}} = 7.5773 \times 10^3 \text{ m/s}$$

For the descent trajectory,

$$v_A = v_{\text{circ}} + \Delta v = 7.5773 \times 10^3 - 152 = 7.4253 \times 10^3 \text{ m/s}$$

$$h = r_A v_A = \left(6.933 \times 10^6\right)\left(7.4253 \times 10^3\right) = 51.4795 \times 10^9 \text{ m}^2/\text{s}$$

$$\frac{1}{r} = \frac{GM}{h^2}\left(1 + \varepsilon \cos\theta\right)$$

At point $A$, $\theta = 180°$, $r = r_A$

$$\frac{1}{r_A} = \frac{GM}{h^2}\left(1 - \varepsilon\right)$$

$$1 - \varepsilon = \frac{h^2}{GM\, r_A} = \frac{\left(51.4795 \times 10^9\right)^2}{\left(398.06 \times 10^{12}\right)\left(6.933 \times 10^6\right)} = 0.96028$$

$$\varepsilon = 0.03972$$

$$\frac{1}{r_B} = \frac{GM}{h^2}\left(1 + \varepsilon \cos\theta_B\right)$$

$$1 + \varepsilon \cos\theta_B = \frac{h^2}{GM\, r_B} = \frac{\left(51.4795 \times 10^9\right)^2}{\left(398.06 \times 10^{12}\right)\left(6.491 \times 10^6\right)} = 1.02567$$

$$\cos\theta_B = \frac{1.02567 - 1}{\varepsilon} = 0.6463$$

$$\theta_B = 49.7° \qquad \sphericalangle AOB = 180° - \theta_B = 130.3° \qquad \sphericalangle AOB = 130.3° \blacktriangleleft$$


© 2007 The McGraw-Hill Companies.